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Editorial Message



Dear Readers,

It gives me immense pleasure to share with you all that IJBAI has successfully come out with the second issue of volume four. It's our immense pleasure in developing the synergy of eminent academicians as well as data science practice professionals to consolidate and improve editorial and operational structures along with increased readership. The journal is a forum for exchange of research findings, analysis, information, and knowledge in various areas which include but are not limited to a destination, yet we emphasis on continuous journey.

I am happy to mention that, Professor Arnab Laha of IIM Ahmedabad has his eminent presence with his column "Analytically yours" reflecting upon Functional Data Analysis and described how to embark upon the analysis when data comes as 'curve'. Ms. Madhumita Ghosh of IBM contributed with a business case to articulate how to enhance operational efficiency by effective management in contact center with various analytical steps. You will also find one more paper "Staff Scheduling in A Product Support Centre" related to contact center process. These two comprehends would provide readers an understanding of simple yet value driven applicability of data-science in service oriented process.

We focused on a technique GARCH Model and brought the perspective of its relevance in two different papers, which are of descriptive and predictive in nature. Though data science plays an important role in process, still we can't eliminate the wide application of it in Operations & Consumer insight forte as well. On this note, you will find three papers addressing 'Purchaser Classification', 'Vendor Selection' & 'Rate of Return on power generation'. By dividing the market and customer into strategic segments, businesses can better position products and services to target their specific customers' needs and desires. Better targeting customer's means better returns on marketing investment. Paper on 'Purchaser Classification' addresses effective segmentation & clustering approach. In procurement procedure, Supplier selection is an integral part, which is often affected by multiple conflicting factors and suppliers' information and performances are usually incomplete and uncertain. Analytic Hierarchy Process (AHP) depicted the advantages to make incomplete comparisons to get towards the final priority selection pointers in the paper 'supplier selection'. In the paper 'Hurdle Rate Analysis', readers will find the blend of several steps of unconventional risk detection, tactical discount factor, and relative influencing independent attributes to establish a robust system to determine rate of return.

I am sure that our readers will enjoy and learn a lot from the present issue. Do let us know your wish, suggestions and views to enrich our journal

Sincerely yours,

Madhumita Ghosh

Joint Editor-in-Chief

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Analytically Yours

Analyzing Data Which are Curves - Functional Data Analysis

Arnab Kumar Laha*

In many areas of application, the data comes in the form of a curve or in other words a function. Consider a hotel having a fixed number of rooms. Suppose the hotel starts accepting booking for one year in advance. The information about the number of rooms booked for a particular date (say for 1st April, 2016) would be available for every day in the period 1st April, 2015 to 31st March, 2016. If the number of rooms booked 't' days prior to 1st April 2016 is denoted by $b(t)$ then the curve $\{b(t) : 0 \leq t \leq 365\}$ is called the booking curve for 1st April 2016. Similar curves would be available for the number of rooms booked for 2nd April, 3rd April etc. Thus we would have the data in the form of curves $\{b_1, b_2, \dots, b_n\}$. Statistical analysis of this kind of data is referred to as functional data analysis. One of the simplest questions to ask with booking curves data is about the shape of the average

booking curve. The answer to this is simple: The mean

booking curve is $\bar{b}(t) = \frac{1}{n} \sum_{i=1}^n b_i(t)$, $0 \leq t \leq 365$, which

in other words is the point wise average of the booking curves. A careful reader by now would be wondering whether at all it is possible to observe $b_i(t)$ for every t . While theoretically it is possible, but in most practical situations $b_i(t)$ would only be observed at some values of t for example for $t = 0, 1, 2, \dots, 365$. Then how does one get to the values of $b_i(t)$ for other values of t ? The answer of course is through interpolation. One of the simplest approach is to do piecewise linear interpolation. In this method we simply connect the observed points by drawing straight lines between adjacent points. Technically, if one has observed $b_i(u)$ and $b_i(v)$ where $u < v$, then the value $b_i(t)$ for an intermediate point t between u and v is

$b_i(t) = b_i(u) + \frac{b_i(v) - b_i(u)}{v - u}(t - u)$. Linear interpolation is

easy to carry out and it gives a curve which is continuous. However, this curve may not be differentiable at the

observed points. This leads to some analytic difficulties which prompts search for more complicated interpolation techniques. An elegant solution is obtained if one uses cubic splines for interpolating instead of the straight lines. In cubic spline interpolation a polynomial of degree three is used instead of the straight lines. This results in a curve which is twice differentiable and has many good properties. Thus cubic spline interpolation is often used for "reconstructing" the functions b_i from the observed values.

In Figure 1 below we give plots of 34 booking position curves for a given class of a long-distance train plying between two major cities of India. Each booking position curve gives the booking position for a train at a certain fixed time of the day starting from 60 days prior to the date of departure to one day prior to the day of departure. The negative booking position indicates that no reserved seat tickets are available and only Reservation Against Cancellation (RAC) or Waitlisted tickets are available.

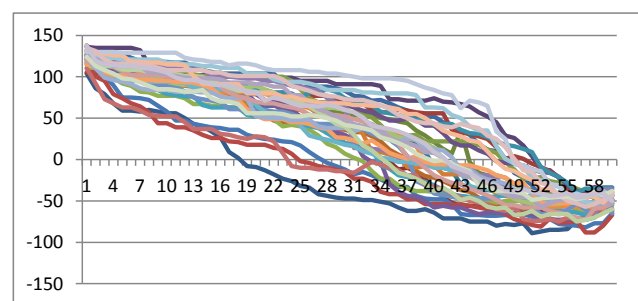


Fig. 1: The Booking Position Curves

The mean curve $\bar{b}(t)$ is calculated by taking the pointwise average of all the 34 curves. By pointwise average we mean that for every value of t we compute

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$\bar{b}(t) = \frac{1}{34} \sum_{i=1}^{34} b_i(t)$. Now we can also compute SD(t), the

standard deviation at each point t. In Figure 2 below we give the plot of the mean curve (denoted as AVG) along with curves $\text{AVG} - 2 \text{SD}$ and $\text{AVG} + 2 \text{SD}$. If the booking position at time t is assumed to follow a normal distribution then approximately 95% of the booking positions on day t will lie within the interval $(\text{AVG}(t) - 2 \text{SD}(t), \text{AVG}(t) + 2 \text{SD}(t))$. An examination of Figure 2 indicates that if a booking is sought to be done up to five weeks before the day of departure of the train there is a high chance of obtaining an instant reservation whereas if the booking is done within the last week before the day of departure of the train there is very little chance of obtaining an instant reservation. Figure 3 below gives the probability of booking position being positive as a function of the number of days since the beginning of the booking.

In Figure 4 below we examine the standard deviation of the booking position as a function of the number of days since the beginning of booking. We find that the standard deviation is not constant over time. In fact the standard deviation of the booking position in the fifth and sixth weeks after the beginning of the booking is much higher as compared to that in the first or last week. Thus any modelling of this data needs to take care of this fact.

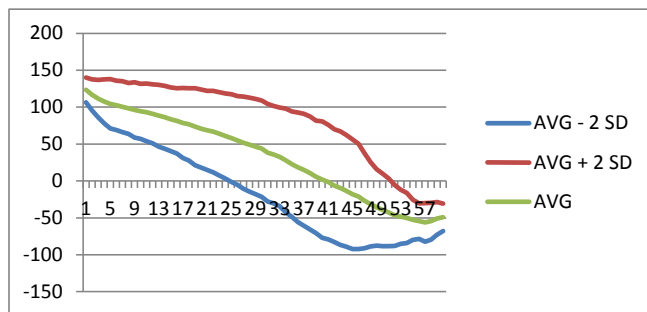


Fig. 2: The Average Booking Position Curve Along With the Curves Signifying the +/- 2 SD Deviations from the Same

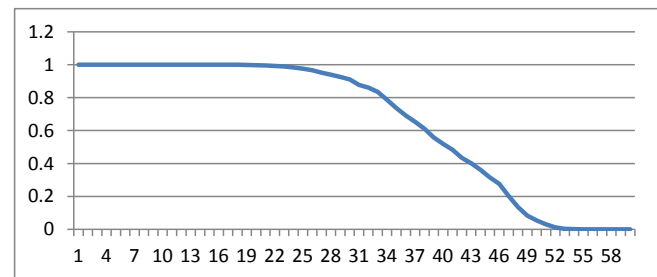


Fig. 3 : The Plot of Probability(Booking Position > 0) as a Function of Number of Days Since Opening of Booking

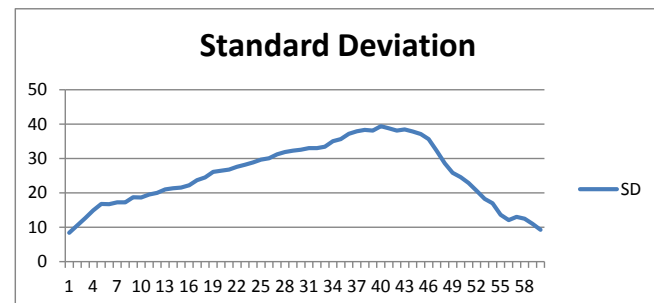


Fig. 4: Variation in the Standard Deviation of the Booking Positions over Days since Booking Opens

One of the key features of functional data is that observations at time points which are very close are often strongly correlated. This is expected because of the tangent line approximation which states that for small values of h , $f(t+h) \approx f(t) + hf'(t)$ where $f'(t)$ denotes the derivative of f at time t . Because of the presence of near perfect correlation between nearby time points the variance-covariance matrix, which is a key tool for describing variation in the multivariate context, often turns out to be ill-conditioned (near-singular). The Figure 5 below gives the correlation between the booking position on day t and the same on day $t+1$. It can be seen from the figure that for most values of t the correlation is very strong (greater than 0.95)

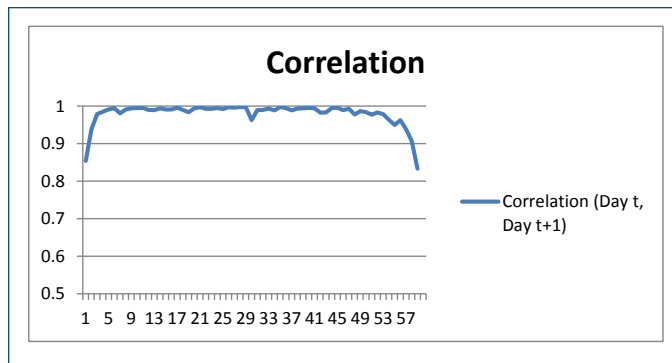
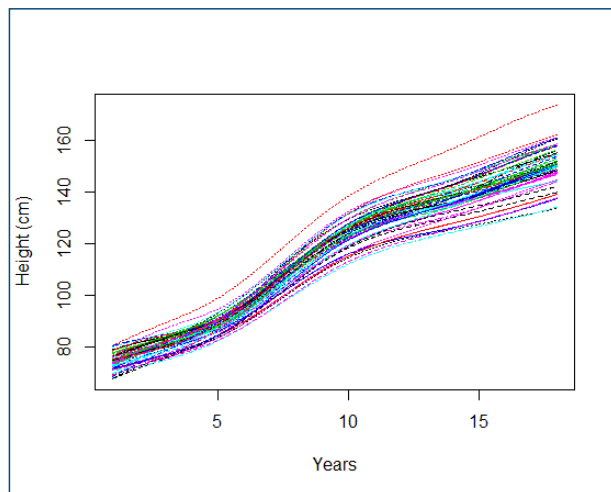
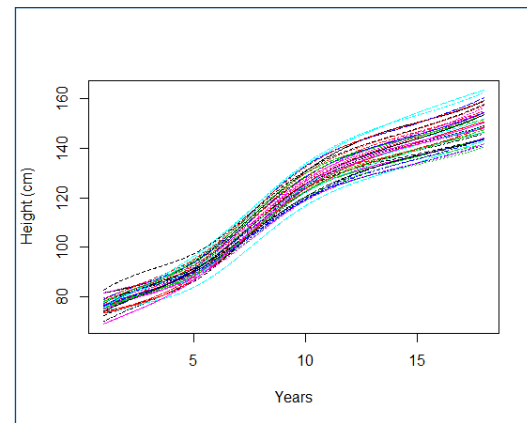


Fig. 5 : The variation in the correlation between the booking positions on day t and day $(t+1)$ as a function of number of days since booking opening.

As a second example let us consider the data from the Berkeley Growth Study. In this study the heights of 54 girls and 39 boys were recorded at 31 different time points between the ages of 1 and 18 years (Tuddenham and Snyder, 1954). Figures 6(a) and 6(b) give the smoothed curves obtained using this data. One of the natural questions that comes to our mind is whether there is difference in the growth pattern in the two groups of male and female children.



(a)



(b)

Fig. 6: (a) Growth curves of the females (b) Growth curves of the males

Figure 7 below gives the plot of the average height of the male children (average_m) and female children (average_f). From the plot we see that the boys and girls have similar heights up to age 13 and from then onwards the boys become taller than the girls on average. Figure 9 gives the standard deviation of the heights of boys and girls at different ages. It is interesting to see that the variation in the heights of the girls exceeds those of the boys till about age 13 but after that the variation in the heights of boys becomes larger. After age 16 the variation in the heights of the boys and girls become almost equal.

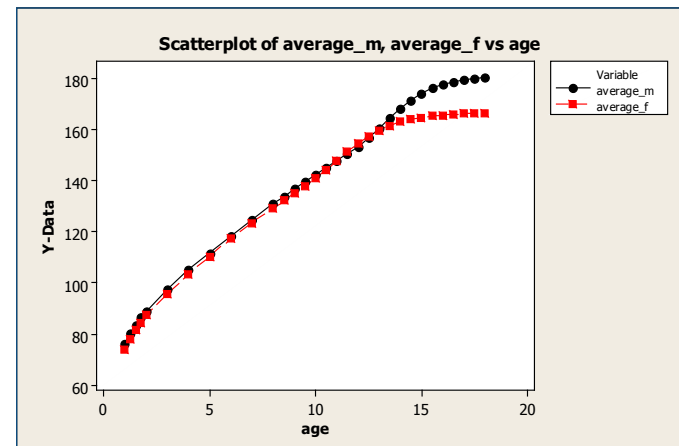


Fig. 8: Average Height of Boys and Girls at Different Ages in the Berkeley Growth Study

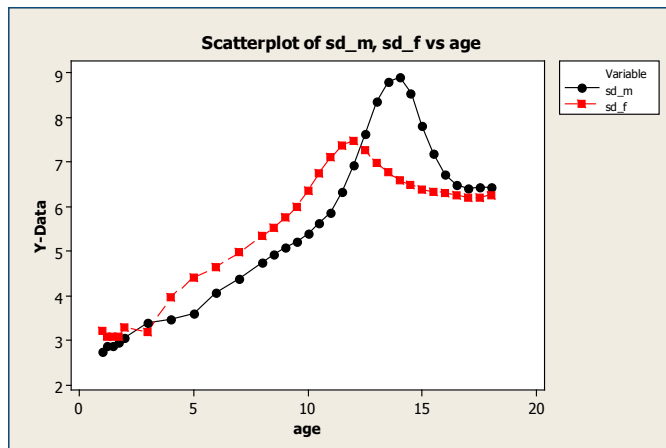


Fig. 9: The Standard Deviation in the Heights of the Boys and Girls at Different Ages

Readers desirous of learning more about Functional Data Analysis from an applied perspective may look at the book by Ramsey and Silverman (2002).

Acknowledgement: Ms. Poonam Rathi, Indian Institute of Management Ahmedabad collected the booking position data which has been used in this article.

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Case Study

The 3i Contact Management Informed - Intelligent - Insightful Led Execution Towards Enhanced Customer Interaction & Experience

Madhumita Ghosh*

Using Predictive Analytics to improve customer satisfaction for a leading Consumer Brand

Enhanced Reputation Management for Brands by Identifying Customer's agony level

Introduction

Effective and efficient way of complaint management is an integral part of enhancing customer experience. Apart from the potential loss of customer and Brand reputation damage, research reveals an average cost of about \$250 to \$350 per complaint in terms of manpower, infrastructure and compensation.

To control this cost and enhance facilities, service led business houses must log and investigate all QRC (Query, Request & Complaint), calls and with special emphasis on complaints effectively, taking steps to enhance service and prevent future complaints. However many processes struggle with this. Complaints are often recorded manually or duplicated across multiple systems making analysis extremely difficult. Some are not logged with proper case description whereas some complaints are misunderstood by staff, leading to errors in the handling process - or the complaint getting discounted entirely. The complaint trail and customer's log in other touch points is also in isolation while considering the importance of the case. These hiccups also lead to taking long time to getting resolved and too many interferences.

This issue needs to be tackled for the sake of costs, reputation and ongoing customer service.

By decoding the crux of 'communications' through path and root-cause analyses of interactions, the service provider can better understand the voice of the customers and understand their psyche.

With the help of 3iContact Management, service providers can:

- Follow a customized approach to quick complaints resolution and overall management, based upon 'case segmentation' and business intelligence
- Increase efficiency of the handling process, through improved workflows, efficient resource utilization with intelligent technology entrenches.

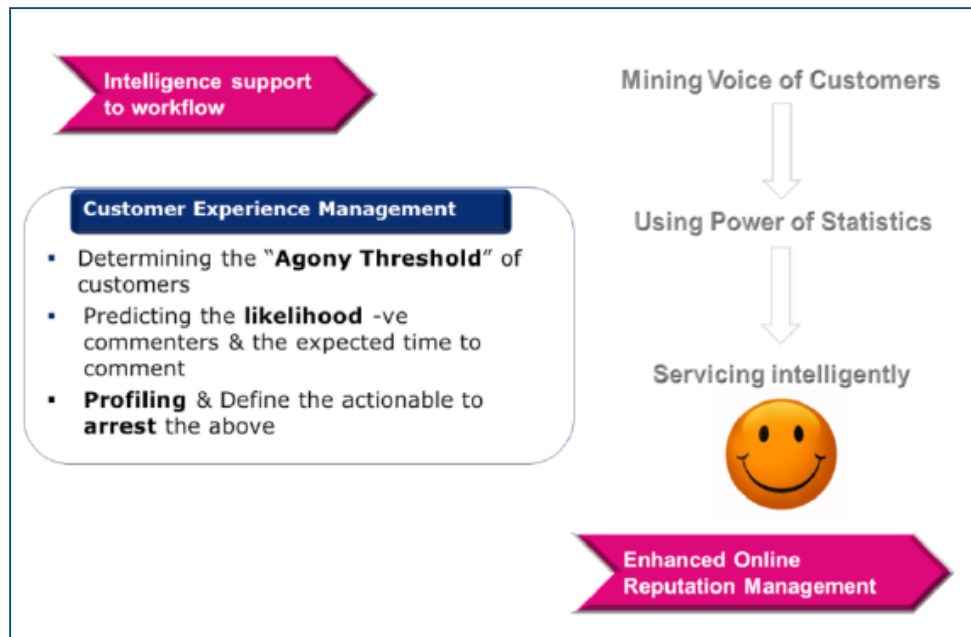
Business Situation

Monitoring social media sites is very important for any B2C brand management strategy. They used online forums and consumer complaint sites to monitor negative feedback about its brands. As digital media's broad reach is significant; it magnifies the audience, brand gets concerned that any negative comment could lead to loss of present and potential clients. While many companies in the retail space focus on responding to negative comments by treating comments as a customer service issue, our client went one step further by focusing on reducing the number of negative comments overall.

Historically the client monitored and extracted comments only from its own sites and sent them to the customer service team to address rectification. The customer service team issued trouble tickets, which were routed to specific departments that worked on resolving the customers' issues. This process seemed to be flawed for two main reasons :

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1. Consumer feedback on resolved issues was never carried back to the sites wherefrom the complaint originated. The negative comment remained alive for few days till no resolution from the company, or positive feedback on the resolution from the customer appeared.
2. The internal analysis (before model deployment) practice was largely focused on operational KPI measurement only, yet ignoring more important insights about customer's 'mindset'. These insights, if identified, could provide guidance for the proactive management of similar complaints and further process improvements to reduce the source of the customer dissatisfaction.



Solution Approach

The solution was worked out to assist our client in finding better process for collecting and addressing specific issues and also analyze the consumer data to provide insights for an improved customer experience.

The solution used an analytics driven approach to understand the different consumers behaviour dimensions that were the key drivers of negative comments about their brand. The solution addressed three major items as mentioned below:

Noise Extraction

Using crawler, the consumer postings were extracted, not only from the client's website and social media pages, but also aggregated posts from other sources including social networking sites, consumer blogs, forums and review sites.

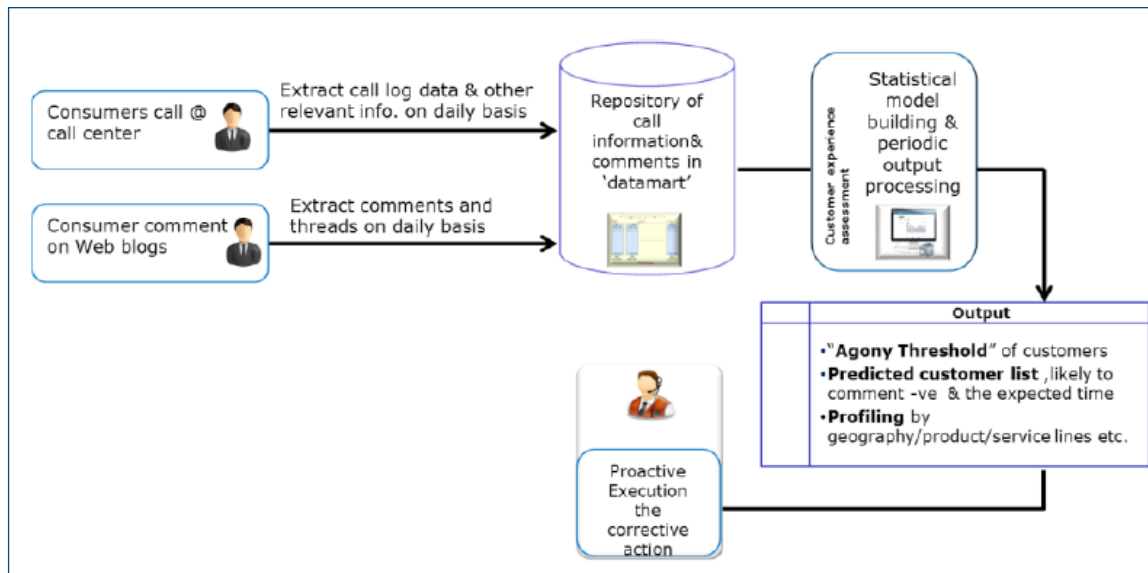
Data Collection and Warehousing

Information was collected, collated and tabulated relating to consumer engagement in our data mart including sales data, customer interaction history and customer details.

Key Driver Analysis and Trending

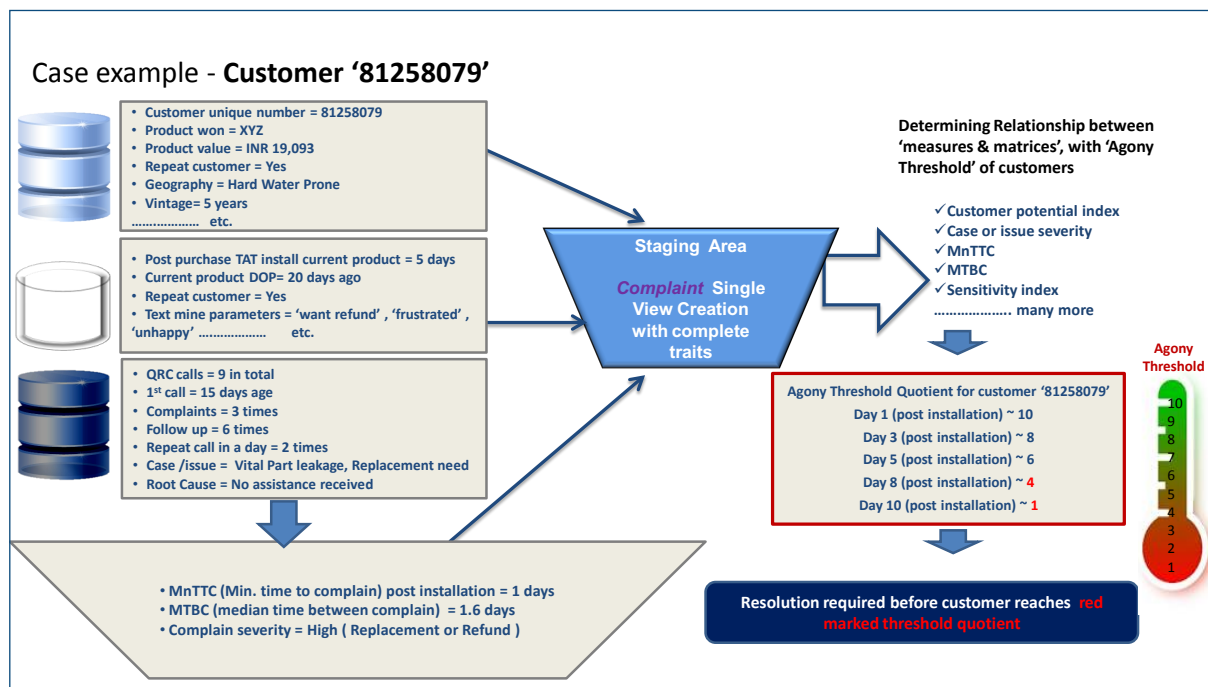
Descriptive analytics model was applied to the data which *identified key trends and patterns* of consumer engagement. As a next step information was segmented by category including geography, product, nature of complaint and life stages, determined consumer behavior dimensions that were key drivers for the negative comments. A positive comments key driver analysis was also done to judge indicators towards positive sentiments.

Using these drivers, a predictive model was built to determine the "Agony Threshold" of their customers, or the point at which they would become dissatisfied enough to lodge a complaint.



The model was deployed every day on a freshly mined dataset. This generated two lists of customers, those with negative sentiments and those with positive sentiments.

The client approached those with negative sentiments. The model was recalibrated once every quarter to align with changed variables like key drivers or degree of influence.



Major Results

Once successfully identified and implemented new processes that addressed our client's gaps and took proactive measures to ensure customer delight. By creating the analytical models which were necessary to predict Agony Threshold, the clients strengthened their relationships with their customer base. Key results include:

- >15% reduction in -ve brand comments month over month and increase in brand imagery scores. Proactive engagement with customers with negative sentiments and addressed wherefrom the complaint originated.
- Reduced complaint resolution time from 3 days to a few hours.
- Proactive interaction with customers to inquire and resolve issues related to products and services – customer satisfaction scores increased.

Identification of key complaint drivers helped in establishing a continuous improvement methodology to proactively improve gaps in products or services before the customers had a negative experience.

The solution provided the benefits as:

- Transformation of interaction approach towards cognizant based rather from SLA based
- Reduce the cost of complaints, with fewer complaints and more efficient handling leading to quick resolution.
- Proactively managed complaints and handling strategies, for improved business performance and reduced complaints in future and resulting in enhanced customer experience.
- Enhanced Customer experience hence retention

Staff Scheduling in a Product Support Centre

Ajith Kumar J.*

Abstract

The present study addresses the problem of staff scheduling in the product support centre of a large multinational technology company in India. Essentially, the problem was to develop rosters that minimise the cost of staffing, while ensuring that service level constraints are satisfied. Broadly speaking, previous research on call centre scheduling has taken two approaches to address the problem, the split approach and the integrated approach. The former handles the full problem by sequentially solving two separate sub-problems – staffing and scheduling – taking the solution to the staffing problem as an input to the scheduling problem. In contrast, the integrated approach proceeds iteratively by solving both parts repeatedly and converging to an optimal solution. We describe the split approach that we took to solve the problem. Our contributions include the modification of a heuristic used earlier to schedule staff in an airport's immigration centre and applying the modified version in conjunction with discrete-event simulation to solve the staffing sub-problem in the product support centre. Using realistic data, we also demonstrate how the heuristic fits into the larger procedure of solving the full staff scheduling problem.

Keywords: Product Support Centre, Staff Scheduling, Discrete-Event Simulation, Heuristic

Introduction

A product support centre is an organisational unit that a company operates to serve its customers. It consists of a team of service agents trained to respond to customer queries, problems, and other service requests. The motivation for this study came from a staff-scheduling problem faced by a real-life product support centre. In short, the problem was to determine the optimal number of staff that the support centre should employ, and the allotment of those staff into weekly shifts and rosters. Actually, managers needed a method which they could use to solve this problem anytime they needed. This paper presents details of the problem, its positioning in

academic literature, and the method that we took to address it. A useful contribution that emerged from the study was the development of a simulation-based heuristic that can help find the solution.

A product support centre resembles a telephone call centre in some ways. As in a call centre, it continuously receives enquiries and complaints from customers. Trained service agents respond to customers. Both the call centre and the product support centre work 24x7 through the year, and hence need staffing throughout. However, a product support centre also differs from a call centre in some ways. Customers make telephone calls to a call centre and interact with a service agent in real-time. Often, and particularly during busy hours, call centre customers wait in a queue to receive service. If the customer is made to wait too long, she may renege (abandon) the call and may or may not call again. On the other hand, customers of a product support centre either submit their enquiries online through a web-portal or send them through email, and wait for a response. As such, they do not interact with a service agent directly. They do not personally experience waiting in a queue and can attend to other work while their enquiry is processed by the product support centre. The situation of a product support centre customer leaving the system (e.g. withdrawing an enquiry), before the centre attends to her request, is rare. Thus, while customer time spent waiting in queue is an important performance parameter for a call centre, the total time elapsed (waiting time + processing time) in completing the response is more important in a product support centre.

Background

This study addressed the issue of staff-scheduling in a product support centre belonging to a large multinational company, hereafter referred to as 'ERP Inc'. ERP Inc offered enterprise resource planning solutions to businesses worldwide. The product support centre was located in Bangalore, India. It attended to the company's cloud-based solutions customers, which were mainly small and medium-sized businesses located across the globe.

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ERP Inc's customers submitted online queries and complaints pertaining to the product through a dedicated web portal and awaited the company's responses. Each such customer incident generated a ticket, or in more technical terms, an 'arrival' to the support centre. In practically all cases, each arrival was processed by one service agent. Since arrivals happened around the clock, the support centre made service agents available 24x7 in three shifts of eight hours each.

ERP Inc's contracts with customers invoked service level constraints (SLC) that dictated how long the support centre could take to process an arrival and give a satisfactory response to the customer. An SLC was expressed in terms of the proportion of arrivals that must be processed within a specified duration of time after entry to the system. Such an SLC is known as the telephone service factor in call centres (Robbins & Harrison, 2010). All arrivals, however, were not of the same type. ERP Inc. classified its arrivals into multiple types based on the perceived level of importance and associated urgency, and assigned a priority level to each type. Normally, arrival types that had faster turn around requirements were considered more urgent and given higher priorities. As a result, a separate set of SLCs was specified for each arrival type.

The SLCs in our problem were global SLCs that were 'horizon-based' and not 'period-based'. This means that they were assessed over the entire scheduling horizon, a duration that spans several periods (such as days or weeks), and not separately for each time period. A discussion on different types of SLCs can be referred in the paper by Robbins and Harrison (2010). These authors note that outsourcing contracts often specify global SLCs. Other studies that have taken the global service level perspective include Cezik and L'Ecuyer (2008) where the service levels are expressed as functions of the staffing for a fixed sequence of random numbers driving the simulation. An optimal solution of this sample problem is also an optimal solution to the original problem when the sample size is large enough. Several difficulties are encountered when solving the sample problem, especially for large problem instances, and we propose practical heuristics to deal with these difficulties. We report numerical experiments with examples of different sizes. The largest example corresponds to a real-life call center with 65 types of calls and 89 types of agents (skill groups and Koole and van der Sluis (2003).

The SLC was most crucial among the product support centre's performance criteria. A simplistic way that ERP Inc. could ensure meeting all SLCs was to employ, train, and assign a large number of service agents to each shift in its operations. However, while abundance in staffing can preempt violations of SLCs, it can also create redundancies in the form of excess manpower in some shifts (if not all) and in turn, high employment costs. Naturally then, the company wanted to have the fewest possible agents in a shift that would ensure meeting all SLCs. The problem, thus, is intrinsically one of cost minimisation.

At the time of this study, managers were allotting service agents to shifts in a somewhat ad-hoc manner, while taking into consideration their leave plans and other contingencies. Owing to this, there was a general feeling that their scheduling could have inherent inefficiencies and they sought a systematic procedure that could drive an optimal staff-to-shift assignment.

The problem had yet another aspect: arrivals did not happen at a constant rate over time; their rates varied across shifts. While some shifts in the week received a large number of arrivals, others had much lesser numbers, implying a non-stationary arrival phenomenon. In some cases, consecutive shifts had very different arrival volumes. Such an arrival phenomenon generated at the product support centre, what was termed flexible demand by Ernst, Jiang, Krishnamoorthy and Sier (2004).

We reviewed academic literature to gain insight into methods that previous researchers have developed to address this problem. We then gathered qualitative information as well as numerical data from the support centre before building a simulation model and developing the required scheduling procedure. In the following, we first present a brief overview of pertinent literature. We then describe the methodology used, the model built, the analysis and its findings. We conclude with a discussion on potential ways to extend and generalise the contribution of this study.

Literature Review

We could not locate research that has modeled staff scheduling in product support centres and ours is perhaps amongst the first studies in this domain. However, the extensive work done on the same problem in call centres comes close. The call centre scheduling problem inherently has two parts that can be called the 'staffing

problem' and 'scheduling problem' respectively. Previous research has taken two broad approaches to address the full problem. We call them the split approach (e.g. Bhulai, Koole, & Pot, 2008; Dileepan & Etkin, 2010; Pot, Bhulai, & Koole, 2008) and the integrated approach (Atlason, Epelman, & Henderson, 2004, 2008; Avramidis, Chan, & L'Ecuyer, 2009; Cezik & L'Ecuyer, 2008; Henderson & Mason 1998; Robbins & Harrison, 2010).

The split approach is modular in nature. It first solves the staffing problem, by dividing the scheduling horizon into small time periods, computing (or forecasting from past data) the average call arrival rate in each period, and then identifying the minimum staffing level in each period to ensure that specified SLCs are all met. It then solves the scheduling problem, by using these period-wise minimum staffing levels as right hand sides of the constraints of an integer linear program (ILP), and solving that ILP to minimise an objective cost function. The ILP's decision variables are the number of staff to be allotted to the different roster lines. The optimal solution to the ILP yields both the total staff size and the number of staff that must be assigned to each roster line.

A section of previous research has used analytical queuing models to address the staffing problem (e.g. Green & Kolesar, 1995; Green, Kolesar, & Soares, 2001). Analytic queuing models confer greater computational efficiency and are preferred when it can be reasonably assumed that the system reaches a steady state quickly, within each time period. However, the steady state assumption may not hold in many real-life situations (Henderson & Mason, 1998). Further, owing to the copious nature of arrivals, the optimal staffing in one period depends upon the staffing levels of other periods (Green, Kolesar, & Soares, 2003). Finally, the service level function, which is most commonly specified as the proportion of customers that must be served with a given time – as in our study – cannot be algebraically expressed (Atlason *et al.*, 2004). Here, simulation is seen as a useful methodology that can provide both the flexibility to accommodate the dependencies between staffing in different periods and an ability to assess a service level function that cannot be algebraically expressed (Atlason *et al.*, 2004, 2008).

The integrated approach, first inspired by Henderson & Mason (1998), iteratively shuttles between solving the scheduling and staffing problems. Each iteration first solves an ILP with staffing levels as constraint right hand

sides, and then uses discrete-event simulation to test if the SLCs are met with those staffing levels. If the SLCs are not met, the procedure generates cutting planes– new constraints in the ILP that increase the lower bounds on the minimum – and solves the ILP again to find new (increased) staffing levels. The procedure terminates when some convergence criteria are satisfied with respect to the staffing vector.

Though the integrated approach overcomes some of the shortcomings of the split approach (such as the steady state assumption, the dependence between staffing levels and arrival rates across periods and the complex nature of the service level function), it faces a couple of distinct challenges. Henderson & Mason (1998) note that it is computationally intensive and requires the service level functions to be concave in nature. Though they have defended this assumption as being reasonable, and others (e.g. Atlason *et al.*, 2004) have developed ways to test for concavity as well, there is an inherent lacuna in the procedure if this assumption is violated.

Both the split and integrated approaches have their strengths; yet research has not concluded on one 'best' approach to the problem. In our study, we followed a procedure that is based on the split approach. However, instead of analytical modeling, we used discrete-event simulation to handle the staffing problem, and then invoked the ILP to handle the scheduling problem. Our contributions include the development of a heuristic and its application in conjunction with discrete-event simulation to locate the minimum staffing vector. Before presenting the heuristic, we present a formal articulation of the problem.

Formal Articulation of the Problem

Though our study's problem is very similar to the call centre scheduling problem, it differs in at least two distinct ways. First, in most call centre studies that we examined, all calls have been treated as having the same priority¹. Second, there is typically only one service level constraint that applies either in each period separately, and sometimes over the entire scheduling horizon. In our product support centre, arrivals were of multiple priorities

¹ Research has modeled different calls requiring different skills, however (e.g. Bhulai, Koole, & Pot, 2008; Avramidis, Chan, & L'Ecuyer, 2009). In such models, appropriately routing the calls would be a feature of the model.

and for each priority, multiple SLCs were applied. We incorporate these differences in our formulation.

As we go along, we define a few terms. A *shift* is an eight hour period that includes a half hour break in between. A *roster line* is a pre-defined set of shifts spread across several days, with some days off. Staff members are assigned to roster lines and not directly to the shifts. Possible examples of roster lines are:

- shift 1 on each of five consecutive days beginning Monday and ending Friday, followed by off days on Saturday and Sunday.
- shift 3 on five consecutive days beginning Thursday and ending Monday, with off days on Tuesday and Wednesday.

Let x_j , $j=1,2,\dots,n$ be the number of staff assigned to each of n different roster lines, and let c_j , $j=1,2,\dots,n$ be the corresponding unit staffing cost vector. Let the scheduling horizon be divided in to m equal-sized time periods. The scheduling horizon is the time period for which a staff schedule is developed, for example, a day, a week, or a month. It is common in call centres to have cyclic schedules (or, tours), whereby staff assignments are repeated in each cycle of the scheduling horizon. Our product support centre too was following a cyclic scheduling system.

Let A : a_{ij} , $i=1,2,\dots,m$; $j=1,2,\dots,n$ indicate whether roster line j covers period i or not, that is, $a_{ij}=1$, if roster line j covers period i and 0, otherwise. As such, the vector Ax denotes the actual number of staff realised in time period i .

Let P be the number of different priorities that arrivals to the support centre can have, and let p be their index. For example, if the arrivals are of five different priorities, then $P=5$, while $p=2$ indicates the second arrival category by priority.

Associated with arrivals of each priority there can be multiple SLCs, with each SLC specifying the proportion of arrivals of *that* priority, which must be served within a maximum time. Let S denote the number of SLCs associated with each arrival category and let s be their index². For example, $S=3$ means that there are three

SLCs associated with each priority in the problem, and for arrivals of priority 1 they might be as follows: ($s=1$) at least 0.75 of the arrivals must be served within 20 minutes, ($s=2$) at least 0.90 of the arrivals must be served within 25 minutes and ($s=3$) at least 0.99 of the arrivals must be served within 28 minutes of entering the system.

Let r : $r(p,s)$, $p=1,\dots,P$; $s=1,\dots,S$ denote the matrix of proportion values and t : $t(p,s)$, $p=1,\dots,P$; $s=1,\dots,S$ denote the matrix of time values associated with the SLCs. In the above example, $r_{11}=0.75$, $t_{11}=20$ and so on. In summary, r and t together help define the set of SLCs in the problem and are inputs to the problem. We note that the number of SLCs in the problem is PS .

While r and t together define the desired service levels, the *actual* service levels achieved depend upon three parameters: the actual number of staff available in each period of the scheduling horizon, the priority-wise arrival rates in each period and the rates at which the arrivals are serviced by the staff.

Let λ : λ_{ip} , $i=1,2,\dots,m$; $p=1,2,\dots,P$ denote the mean arrival rate of tickets of priority p in period i , μ : μ_p , $p=1,2,\dots,P$ denote the mean service rate of tickets of priority p and as noted earlier, Ax gives us the actual number of staff realised in time period i . Then, we can use q : $q(p,s,Ax,\lambda,\mu)$, to denote the actual fraction of tickets of priority p that complete their service within time $t(p,s)$. In other words, q is the matrix of actual service levels achieved.

Finally, let d : d_i , $i=1,2,\dots,m$ denote the staffing vector, where d_i stands for the number of staff working in period i . Given these definitions and notations, the full problem and the sub-problems can be articulated as follows:

Full problem	Staffing sub-problem	Scheduling sub-problem
Minimise $Z = c^T x$	Minimised d_i for each i	Minimise $Z = c^T x$
Subject to:	Subject to:	Subject to:
$q(p,s,Ax,\lambda,\mu) \geq r(p,s)$;	$q(p,s,d,\lambda,\mu) \geq r(p,s)$	$Ax \geq d$;
$x \geq 0$ and integer	$d \geq 0$ and integer	$x \geq 0$ and integer

² Here, we assume that the number of SLCs is the same across priorities. This assumption, however, does not restrict the problem since we can take the maximum number of SLCs

across priorities as a universal maximum. For priorities that have lesser SLCs than this maximum, we can create dummy SLCs to fill the gap, having the time limit as infinity (and use a very large number during execution).

The objective of the full problem is to find an optimal \mathbf{x} . However, the service level function \mathbf{q} is nonlinear and is difficult to express algebraically. This makes the full problem an integer nonlinear program and difficult to solve. Following the split approach, we first solved the staffing sub-problem to obtain the minimum \mathbf{d} . We used a heuristic in conjunction with a simulation model of the product support centre to find \mathbf{d} . We took the \mathbf{d} thus obtained as an input to the scheduling sub-problem—which is an ILP—where it served as the right hand side of the ILP's constraints. Solving the ILP yielded an optimal \mathbf{x} . The heuristic to find \mathbf{d} is a novel feature of our study, and we describe it in the following.

The Heuristic

Our heuristic builds upon one used by Mason, Ryan, and Panton (1998) to optimise staffing in the immigration of an international airport. They termed the heuristic Algorithm 1. Algorithm 1 begins by finding an initial feasible solution (\mathbf{d}_0) of staffing levels; a feasible solution is any \mathbf{d} that satisfies all SLCs. For convenience, the initial feasible solution is chosen with equal staffing in all periods. Algorithm 1 then systematically examines each period in turn, reduces the staffing level in that period and checks whether the reduced staffing solution is also feasible. It rejects a reduction that leads to an infeasible solution and terminates upon reaching a dominant minimum. Termination happens when there is no time period left in which a feasible reduction is possible.

Our heuristic differs from Algorithm 1 essentially on the rule for the next period to pick. In Algorithm 1, the next period to pick is the one with the greatest staffing level amongst all periods left in the candidate list. The candidate list initially consists of all time periods in the scheduling horizon. When there is a tie, the heuristic chooses the period with the lowest index number. In contrast, our heuristic picks the next period on the basis of the net arrival rate in the period (across all priorities). This rule is driven by the intuition that periods with lower net arrival rates would need lower staffing than those with higher arrival rates. When there is a tie, we break it on the basis of the net arrival rate of the immediately preceding period.

The formal articulation of our heuristic draws on the one presented by Mason *et al.* (1998, pp. 164), but incorporates the arrival rate vector (λ). We present the same here:

Let $Z = \{1, \dots, m\}$ be the candidate list of all periods in the scheduling horizon.

Let \mathbf{d}_0 be the starting feasible solution.

Set $\mathbf{d} \leftarrow \mathbf{d}_0$

repeat

Let $z = f(\mathbf{d}, \lambda, Z)$ be the index of the next period to reduce.

Consider the solution $\mathbf{d}' : (d_1, d_2, \dots, d_{k-1}, d_k - 1, d_{k+1}, \dots, d_m)$

Check whether \mathbf{d}' is feasible.

If yes then

Let $\mathbf{d} \leftarrow \mathbf{d}'$

else

Remove z from Z , i.e. let $Z \leftarrow Z \setminus z$

Until $Z = \{\}$

Staffing plan \mathbf{d} is a dominant minimum

We experimented with two variations of this heuristic:

- Heuristic 1: begin with the period that has the lowest net arrival rate; then move in the order of increasing net arrival rate, and
- Heuristic 2: begin with the period that has the highest net arrival rate; then move in the order of decreasing net arrival rate.

In Heuristic 1, we pick $z = f(\mathbf{d}, \lambda, Z)$ using the rule, $z: \lambda_z = \min \lambda_i (i \in Z)$, whereas for Heuristic 2, it is $z: \lambda_z = \max \lambda_i (i \in Z)$. In both cases, when there is a tie for $\min \lambda_i$, with say $\lambda_g = \lambda_h = \min \lambda_i$ we use the rule, $z: \lambda_z = \lambda_g$ if $\lambda_{g-1} < \lambda_{h-1}$ else $\lambda_z = \lambda_h$ ($g, h \in Z$). That is, to break the tie we take that period, whose immediately preceding period has a lower net arrival rate.

Methodology

ERP Inc's product support centre makes use of an automated internet-based tool known as the Activity Management System (AMS) to manage the arrivals of customer tickets and their processing by service agents. We gathered records of all arrivals that occurred over a period of 91 days (13 weeks, or 3 months). Each record contained the following information about the arrival:

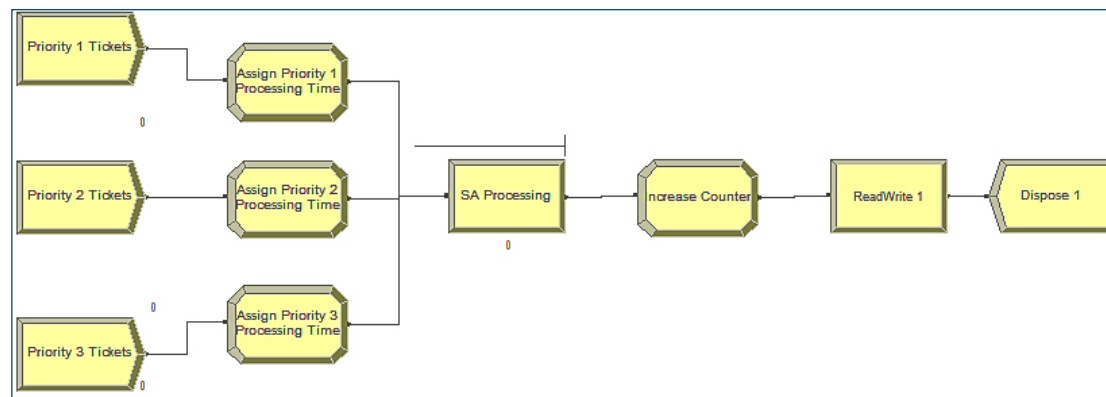


Fig. 1: Arena Model Used in This Study

Table 1: Consolidated Arrival Pattern Used as an Input for Simulation

Arrival Priority	Time period	AVERAGE NUMBER OF ARRIVALS PER HOUR						
		SUN	MON	TUE	WED	THU	FRI	SAT
1	06:00 to 14:00	0.79	0.19	0.21	1.00	0.67	0.55	4.50
	14:00 to 22:00	0.03	0.60	0.66	2.11	1.21	2.12	3.72
	22:00 to 06:00	0.89	0.21	0.47	0.48	0.70	0.08	2.23
2	06:00 to 14:00	2.11	6.19	7.15	8.26	8.26	8.27	2.20
	14:00 to 22:00	2.08	5.23	5.48	6.63	5.24	4.88	1.98
	22:00 to 06:00	0.88	1.54	3.09	3.79	3.10	2.89	1.63
3	06:00 to 14:00	0.12	0.27	1.05	0.90	0.56	0.45	0.01
	14:00 to 22:00	0.16	0.29	0.81	0.50	0.51	0.33	0.13
	22:00 to 06:00	0.14	0.20	0.82	0.71	0.71	0.48	0.08

its priority level, the date and time of its creation in the AMS, the date/time at which it was picked by a service agent, and the date/time at which the agent closed it and it left the system. In all, our data set had 13349 records. Some records had defective data such as missing values or extremely high and unrealistic numbers, suggesting erroneous entries. After cleaning, the dataset reduced to 13219 records, of which 2471, 9784 and 964 records belonged to ticket priorities 1, 2 and 3 respectively.

To apply the heuristic and solve for the d , we built a discrete event simulation model of the system using Arena for Windows (version 10.0), as shown in Fig. 1. Using the data set, we prepared three essential inputs to the simulation model – the arrival pattern, the service time distributions and the staffing vector.

First, we will discuss the arrival pattern. The service agents work in 8 hour shifts that run 06:00–14:00, 14:00–22:00 and 22:00–06:00. We computed the net average arrival rates (priority-wise as well as overall) within

each shift and found that shift-wise rates were somewhat repetitive from one week to the next. Based on this, we decided to adopt a *one-week* scheduling horizon. Treating each 8 hour shift as a distinct time period yields 21 time periods in the scheduling horizon. We consolidated the arrival rates into these time periods, and specified arrivals as the average number within each time period, for each priority category, on each week-day.

Table 1 presents this consolidated arrival pattern data. For example, on Sundays, 2.11 tickets of Priority 2 arrived on an average from 06:00 hours to 14:00 hours, on Thursdays, it was 0.56 tickets of Priority 3 in the same period, and so on³.

³ ERP Inc did not object to our publishing this study and its contributions, but requested us to conceal the original data. Accordingly, we have modified the key data. More specifically, the arrival rate, service time and SLC data presented here are not the original values. However, the numbers presented here are realistic and representative of those in real-life. It is important to note that the changed data affects neither the

Table 2: Processing Time and the SLC Specifications

Arrival Priority	Processing Time (min)	SLC	
		85 th percentile (min)	99 th percentile (min)
1	1 + Expo(6.3)	15	40
2	1 + Expo(12.1)	30	60
3	1 + Expo(8.3)	45	120

We assumed that arrival within each shift followed a Poisson process.

Second, we examined the time spent by service agents in processing arrivals of each priority category. Using the Input Analyzer feature of Arena, we identified the probability distributions followed by processing times, separately for each priority. Table 2 presents these distributions. For example, the time spent by the SAs in processing arrivals of priority 1 followed the distribution 1+Expo (6.3) minutes, where Expo (6.3) represents a negative exponential distribution with the mean of 6.3. We also gathered data on the SLCs associated with each priority (Table 2). For example, 0.85 of Priority 1 arrivals (85th percentile) had to be completed in 15 minutes, while 0.99(99th percentile) had to be completed within 40 minutes of the arrival, and so on.

Finally, we discuss the staffing vector. We noted that each 8 hour shift includes a half hour break in between. We learnt from discussions with managers that agents in a given shift do not all take the break at once. Rather, about half of them take the break first and the rest take it after the first group returns. This ensures that some arrivals are always being processed. We incorporated this in the simulation model as follows. If d_i is the number of staff assigned to shift i in the model, then the staffing level is d_i for the first three and a half hours, $d_i/2$ for the next one hour and again d_i for the remaining three and a half hours of that shift.

We validated the simulation model by running 30 replications, each for a period of 91 days (same as the data gathered), keeping the staffing levels similar to those currently practiced. We compared the number of arrivals generated and the average time durations⁴ resulting from the simulation model, with those currently experienced

in practice. While the number of arrivals matched very closely, we found acceptable similarities in the durations.

We then performed what-if analyses on the staffing vector with each of the two heuristics described earlier. We first found a ‘ceiling vector’ \mathbf{d}_0 , which, as explained earlier, is an initial feasible solution from which the heuristic begins its descent towards a dominant minimum. For this, we began by assigning exactly 1 service agent to each period, that is $\mathbf{d}_0 = (1, 1, \dots, 1)$ and ran the simulation. When the solution turned out to be infeasible, we increased the staffing level by 1 unit in each period and ran the simulation again. We repeated this until we found the minimum feasible ceiling vector to be (5, 5, ..., 5).

Each iteration of the heuristic involved making a unit reduction in the staffing level in one of the 21 shifts, running the simulation and examining if any constraint was violated. The heuristic examined each of the 21 shifts in turn at least once (but possibly several times), and terminated when a dominant minimum was reached, that is, when no further reduction in staff was possible without violating a constraint⁵. We conducted all our iterations by manually changing the staff levels in the Arena model. In each iteration, we ran 30 replications of the simulation for 91 days, same as the duration of the data. After reaching the dominant minimum, we ran the second-stage ILP, to compute the minimum team size needed if that staffing plan was followed. Since our problem was not large, we could do this using MS Excel’s Solver. We took cost per employee as the same across all roster lines; having different costs will not affect the methodology.

methodology/heuristic that we present (which are the focus of the paper), nor the paper’s conclusions.

⁴ Priority-wise: averaged over all tickets, the time spent in queue and the total time spent in the system.

⁵ Each iteration results in either an acceptable unit reduction in the number of staff, or a constraint violation. When a constraint violation happens, that shift will no longer be examined. This means that the number of iterations to reach the dominant minimum in a given trial of a heuristic, equals the total number of reductions made across all shifts (since one reduction happens in each iteration), plus the number of shifts.

Table 3: Results

Time Period (Shift)	Net Average Arrival Rate (h^{-1})	Ceiling (initial feasible solution)	Optimal d reached by		
			Mason <i>et al.</i> 's Algorithm 1	Heuristic 1	Heuristic 2
Sun 1	1.72	5	4	2	3
Sun 2	1.56	5	4	4	4
Sun 3	0.83	5	4	3	3
Mon 1	4.64	5	4	4	4
Mon 2	4.00	5	4	4	4
Mon 3	1.19	5	4	3	4
Tue 1	5.41	5	4	4	5
Tue 2	4.24	5	4	4	4
Tue 3	2.43	5	4	4	5
Wed 1	6.37	5	4	4	5
Wed 2	5.34	5	4	5	4
Wed 3	2.95	5	5	4	5
Thu 1	6.28	5	5	4	4
Thu 2	4.14	5	4	4	4
Thu 3	2.48	5	4	4	5
Fri 1	6.26	5	5	5	4
Fri 2	4.03	5	5	4	4
Fri 3	2.19	5	5	4	5
Sat 1	2.47	5	4	4	4
Sat 2	2.17	5	4	4	4
Sat 3	1.63	5	4	3	4
Minimum manhour/week ($\sum_i d_i$)*8		840	712	648	704
Variance			0.190	0.429	0.362
Minimum team size needed ($\sum_j x_j$)			18	17	19
Roster lines			14	12	14

Results and Discussion

Table 3 displays the results. It presents the average net arrival rate for each time period (shift), the ceiling values of the staffing plan and the optimal d reached by applying each of the two experimental heuristics and Algorithm 1 of Mason *et al.* (1998). Figs. 2-4 display these d respectively. Table 3 also displays for each d , the minimum man hours/week it implies (that is, $8 \cdot \sum_i d_i$), the variance in staffing levels across time periods within that d , the optimal team size (that is, $\sum_j x_j$) needed to achieve that d , and the number of distinct roster lines that the team will be scheduled into, which is the same as the number of non-zero x_j in the x .

Heuristic 1 leads to a markedly lower weekly man hour requirement (648) than those of Heuristic 2 and Algorithm 1 (704 and 712 respectively). Clearly, the optimal d reached by solving the staffing sub-problem depends upon the heuristic deployed. Heuristic 1 also yielded the best team size (17) vis-à-vis those of Heuristic 2 (19) and Algorithm 1 (18).

The Role of Variance

We make a couple of observations. The optimal team size resulting from Heuristic 1 is only slightly better (lower) than that obtained by Algorithm 1 (17 vs. 18) even though Heuristic 1 led to markedly lower weekly man hour requirement (648 vs. 712). Further, the optimal team size

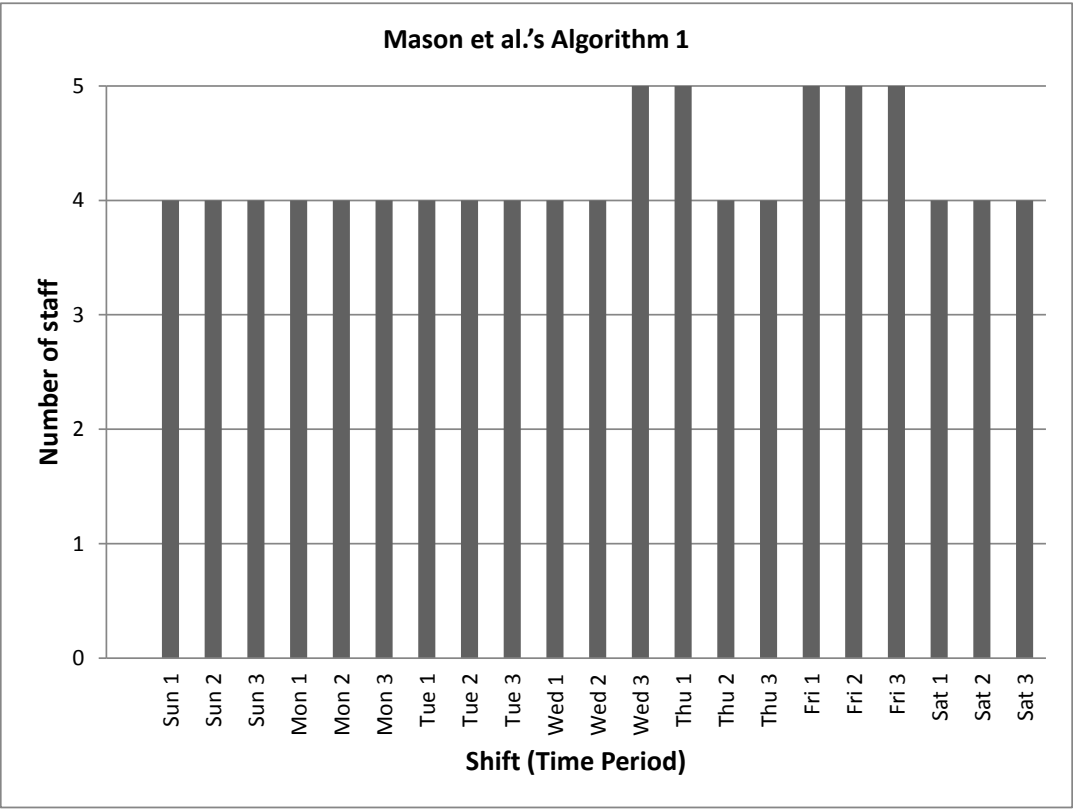


Fig. 2: Staffing Solution Obtained by Mason, Ryan, & Panton's (1998)Algorithm 1

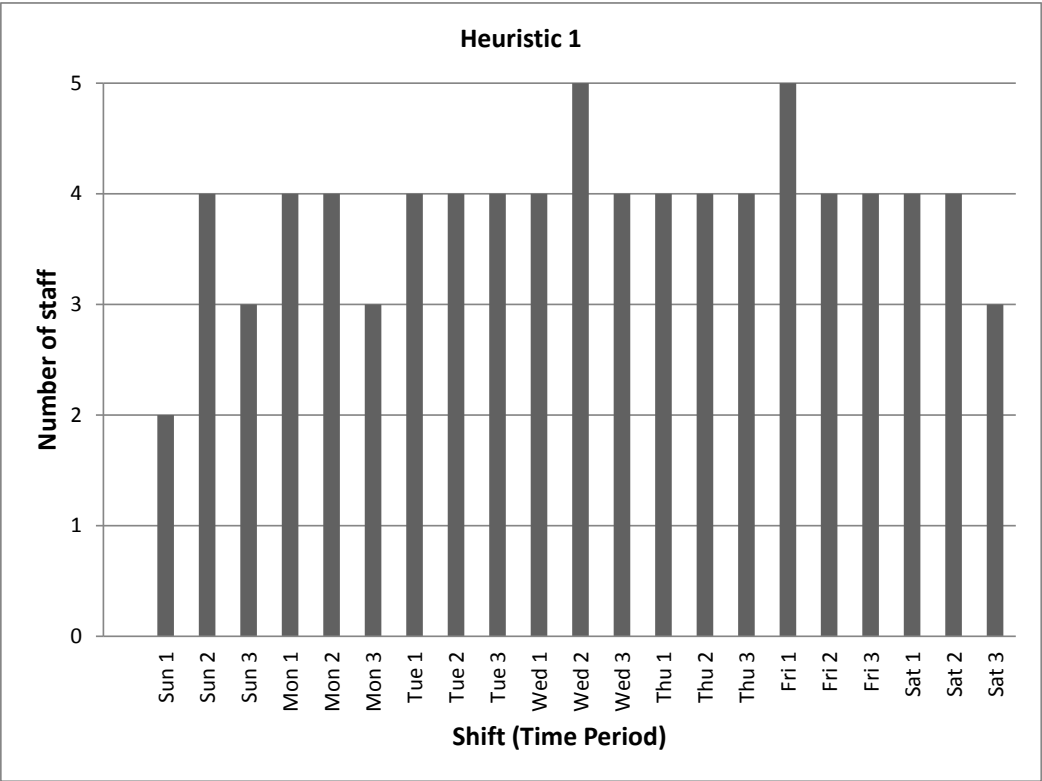


Fig.3: Staffing Solution Obtained by Heuristic 1

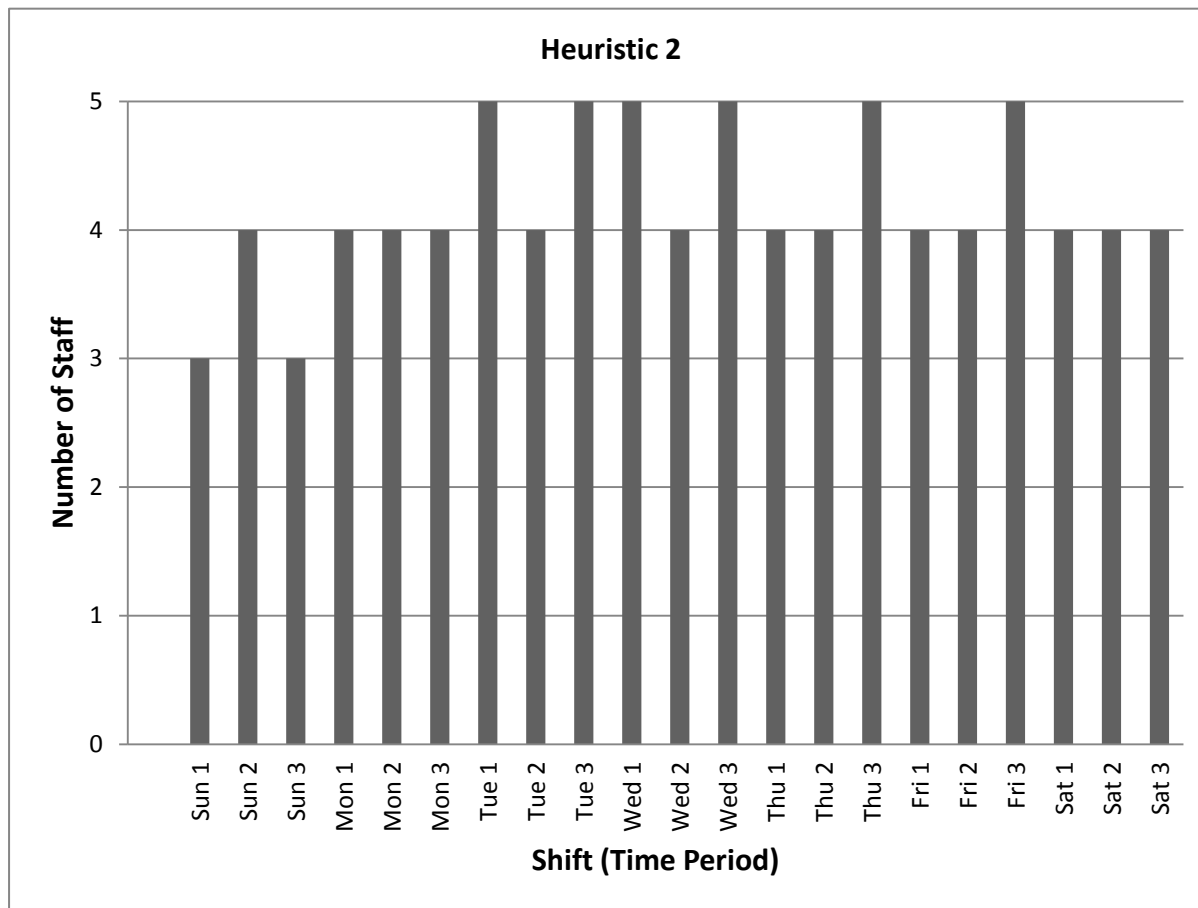


Fig. 4: Staffing Solution Obtained by Heuristic 2

resulting from Heuristic 2 is one unit worse (higher) than that obtained by Algorithm 1 (19 vs. 18) despite Heuristic 2 having better (lower) weekly man hour requirement (704 vs. 712). These observations can be explained by the variance of the period-wise staffing values (d_i) within the \mathbf{d} . The solution obtained by Heuristic 1 has a variance of 0.429, while that from Heuristic 2 has a lower variance (0.362) and that from Algorithm 1 has the lowest variance (0.190). We recollect that the d_i values become right-hand sides of the set-covering ILP in the second sub-problem. When the variance of d_i is more, the ILP needs to use a larger set (team) to provide the required minimum cover to all periods. More generally, we can expect that given two staffing solutions with equal weekly man hour requirements, the solution with lower internal variance is likely to lead to a lower team size. This means that the effectiveness of the staffing heuristic should be judged not only by the total weekly man hour requirement that it leads to but also by the variance in its staffing across periods. Naturally, a heuristic that gives us both the least weekly man hour requirement as well as the

lowest variance across shifts can be expected to give us the lowest team size.

Number of roster lines

The last column of Table 3 shows the number of distinct roster lines that emerge from the ILP's solutions. As noted earlier, this is the same as the number of non-zero x_j in the optimal roster schedule \mathbf{x} . From the perspective of managerial convenience in scheduling, having lesser roster lines (greater parsimony) might be easier to handle. Again, Heuristic 1 does best with 12 distinct lines vis-à-vis the 14 lines that Heuristic 2 and Algorithm 1 yield.

Algorithm 1 vs. Heuristics 1 and 2

There seems to be an inherent limitation in Algorithm 1, which our heuristics attempt to overcome. While selecting the next period to reduce staff, Algorithm 1 picks that period in the candidate list with the largest number of

staff in it. When there is a tie, it picks the chronologically earlier period. Given that the initial feasible solution has equal staffing in all periods, Algorithm 1 begins with the first chronological time period and proceeds by selecting periods chronologically. As a result, it can induce lower staffing levels in earlier time periods of the scheduling horizon and higher ones in later periods. Though, this may seem problematic, Mason *et al.*'s (1998) study deals with scheduling customs staff in the departure process of an international airport, and the authors propose that such biasing is advantageous as it gives, "... a more robust solution should delays occur" (pg. 164).

While adapting Algorithm 1, we recognised that a product support centre differs from an international airport in some ways. Though demand is flexible in both cases, the scheduling horizon for us is not just a day as in the airport; it could be a week, ten days, or even longer, depending upon how arrival patterns repeat themselves over time. Consequently, the bias that results from picking chronologically earlier periods first can be disadvantageous as it can arbitrarily restrict the final solution to the scheduling problem from reducing below a certain point. In other words, the sequence in which the periods are picked for reduction can matter to the dominant minimum reached. As seen, Heuristic 1 performed distinctly better than Heuristic 2 (Table 3).

In modifying Algorithm 1 to arrive at our heuristic, we were aware that our guiding intuition that shifts with lower arrival rates will need lower staffing cannot be generalised owing to dependencies between periods. Arrivals that occur in a given period may 'spill over' and be processed by staff in subsequent periods and this is typical for arrivals that occur close to the end of a given period. As noted earlier, the staffing level in one period influences service levels achieved in periods that immediately follow it. For this reason, even though it is possible to compare heuristics with each other and identify one that is better than others, it is difficult to firmly ascertain whether we actually reach the optimal solution by either heuristic. Alternate heuristics may exist that can yield better minima and lower cost rosters. Future experiments can examine other variations of these heuristics. For example, one variation could be to choose the next period to pick as the one with the lowest arrival rate for the highest priority arrival and yet another could be with the lowest arrival rate for the highest priority arrival.

All the three heuristics compared in this study work by reducing one staff at a time in a given period, before moving to the next period. The distinctive feature of Heuristic 1 is that it first chooses periods that have lower average arrival rates. This seems to be its strength. We explained all three heuristics to the company and showed them the differences in results.

Conclusion

The purpose of our study was to find a method for ERP Inc. to determine the optimal number of staff that the support centre should employ, along with the scheduling of those staff into rosters. Did our efforts serve that purpose? The answer is: yes, but in a more fundamental way than the numerical results that our method provides. The significant positive that managers at ERP Inc perceived, stemmed from the realisation that they now had a scientific procedure to justify the number of staff they employ and the rosters they develop. They no longer had to depend only on the intuitive and ad-hoc procedure they had been following all along. In the year-long period since this study concluded, the product support centre team has regularly used the split procedure that we discussed in this paper, and are satisfied with it. The procedure has provided a scientific support for intuition and common sense in actual practice.

From an academic perspective, several research studies have examined the scheduling problem in the call centre domain. However, our study has certain novel features. It is perhaps amongst the first studies on staff scheduling in product support centres. As such, it incorporates multiple arrival priority categories and multiple service level constraints within each category, something not generally seen in the call centre scheduling research. It also offers a simulation-based split approach to the call centre scheduling problem along with a heuristic not presented earlier. Thus it makes a useful contribution to research on scheduling. Notwithstanding this, more work is necessary to understand how our procedure compares to competing procedures in terms of efficiency. Indeed, we believe that finer insights and stronger conclusions are possible, by repeating the above trials with multiple datasets.

During this study, we manually changed staffing levels in the model, while applying the heuristic. As a result, each trial of the heuristic took several hours to conduct. Nonetheless, the exercise gave us a glimpse of how the

heuristics work and helped us gain some early insight into why one heuristic may be better than another. Going forward, we are exploring the possibility of developing a code that can automate the entire heuristic descent and give us the dominant minimum with the press of a key. We expect that run time would reduce to a few minutes, and make it practical for us to conduct an exhaustively large number of trials within a few weeks. Armed with this code, we intend to experiment with several sets of input data (arrival rate, processing time, SLCs) and compare the performances of the heuristics with each other. We feel that that such work can lead to notable contributions to research on staff scheduling in call centres and product support centres.

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Volatility Behaviour in Emerging Stock Markets – A GARCH Approach

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Abstract

This study primarily focuses on three aspects - (i) volatility in the emerging stock markets across globe by application of GARCH family models, (ii) study of ARMA structures, and (iii) a comparison of symmetric and asymmetric volatility. In the last decade or so, investors from developed countries are mostly focusing on the emerging economics as their investment opportunities. They associate a good amount of risk premium with these countries as far as the risk and return are concerned with their investments. Investments drawn from developed nations seems to make stock markets of emerging nations more volatile as these investment are exposed to both irrational and rational factors. Hence it's imperative to understand the volatility behaviour of emerging stock markets over a period of time and also to study the comparative analysis of the volatility behaviours' across these markets. This draws us to revisit the topic on volatility behaviour considering the emerging markets for this study. In this paper an attempt is being made to estimate the volatility behaviour of stock markets of 10 emerging economics and hence concentrated on India, China, Indonesia, Sri Lanka, Pakistan, Russia, Brazil, South Korea, Mexico, and Hong Kong.

Keywords: Stock Market Volatility, GARCH, Emerging Markets, Heteroscedasticity, ARMA

JEL Classification: - G1, G10, G18, G19

Introduction

Stock market volatility has always received a lot of attention in the empirical literatures; however research on this topic still has the potential to explore new information. Hence this study is an attempt to explore the volatility pattern of ten emerging markets selected on the basis of their listing as emerging economics. Investors,

regulators, and other stakeholders always keep an eye on the volatility of different stock markets to draw their decisions. A lot of volatility had been witnessed during the phase of global economics crisis during 2008-09 and Euro Zone debt crisis during 2010-13. This study thereby given emphasis on these two periods and hence considered a total period of almost eleven years (2003-2014) to estimate the volatility of the countries considered for this study.

Literature Review

Yang and Liu (2012) studied the forecasting power volatility index in emerging economics particularly in reference to Taiwan market and found that the volatility index(TVIX) of Taiwan stock index options is a strong indicator of future stock market volatility. The TVIX outperforms the historical volatility and the GARCH volatility forecast in assessing the activities of Taiwan's stock market. Kiymaz and Girard (2009) studied stock market volatility and trading volume in emerging market and found that persistency of conditional volatility is high and very close to unity, implying that current information can be used to predict future volatility. Unexpected news doesn't affect volatility significantly but the forecastability of volume activity is high. Franses and Dijk (1996) forecasted stock market volatility using (nonlinear) GARCH model and as per their findings the Q GARCH model is best when the estimation sample does not content extreme observations such as the 1987 stock market crash and the GJR model cannot be recommended for forecasting. For their estimation of volatility they used 'within sample estimation' and 'out of sample estimation' and found that the forecasting performance of the GARCH type models appears sensitive to extreme within-sample observations. Gulen and Mayhem (2000) studied stock

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Table 1: Emerging Stock Market at Glance

Country	Name of Exchange	Index	Year of Establishment	Market Capitalisation	Index Calculation Methodology	No of Companies Lists
India	Bombay Stock Exchange	Sensex	1875	USD 1.32 Trillion as of January 2013	Free-Float Market Capitalisation	5000+
China	Shanghai Stock Exchange	SSE Composite	1990	USD 2.3 trillion (2011)	Free-Float Market Capitalisation	998+
Brazil	IBOVESPA	IBOVESPA	1890	USD 1.22 Trillion (2012)	NA	365
Pakistan	Karachi Stock Exchange	KSE 100	1947	USD 53.3 billion May 2013	NA	652
Srilanka	Colombo Stock Exchange	ASPI	1985	LKR 2.3 Trillion	Weighted Market Capitalisation	289
Mexico	Mexican Stock Exchange	IPC	1933	USD 460.4 billion	Weighted Market Capitalisation	466
Russia	Moscow Exchange	RTS	1995	USD 13.5 billion	Free Float Market Capitalisation	1845
South Korea	Korea Stock Exchange	KOSPI	1960	USD 1.1 trillion	Weighted free float market Capitalisation	773
Indonesia	Jakarta Stock Exchange	JSKE	1912	USD 426.78	Modified Weighted Capitalisation	462
Hong Kong	HongKong Stock Exchange	HSI	1891	HK\$16.985 trillion (Nov 2011)	Free Float Capitalisation	1421

index futures trading and volatility in international equity markets and found that symmetric GARCH and GJR GARCH perform marginally better than others. United States and Japan's volatility may have increased after listing of stock index, but for other countries volatility decreased or stayed roughly the same. In most of the other cases, volatility tends to be lower in periods when open interest in stock index future is high, but in case of United States and Japan there is an opposite result. Mala and Reddy (2007) measured stock market volatility in an emerging economy and the study observed that seven out of the sixteen firms listed in Fiji's stock market are volatile. It is further found that change in the interest rates have significant effect on stock market volatility. The finding further suggests that the volatile firms are exposed to government regulations, where the liquidity has been low over the years. Jayasuriya, Shambora and Rossiter (2009) studied asymmetric volatility in emerging and mature markets and the empirical results of the study suggest that descriptive statistics for the sample data shows higher standard deviation for emerging markets in comparison to the mature markets. First sub-period shows high α for emerging markets indicating that emerging markets often react somewhat more to news in

comparison to the mature markets. Whereas, second sub-period shows asymmetric volatility for both mature and emerging markets. And, in the third sub-period each of the mature markets exhibits asymmetric volatility as do the emerging markets with the exception of Philippines. Wei (2002) forecasted stock market volatility with non-linear GARCH models for China and the results of the study suggest that GARCH (1,1) model is preferred within the sample period i.e., from 1992 to 1996 to QGARCH and GJR GARCH models whereas, the QGARCH model is found appropriate in case for forecasting out of the sample i.e., for 1997 and 1998. McMillan and Alan (2003) studied asymmetric volatility dynamics in high frequency FTSE-100 stock index futures and the results suggest that a high kurtosis, both positive and negative skewness, and series is not normally distributed as confirmed by Jarque-Bera test. GARCH effect is in the data at all frequencies that has been confirmed by ARCH-LM test. TGARCH (1,2,1) models, the positive coefficients obtained, indicate negative shocks increase volatility by a greater magnitude than positive shocks of equal size. QGARCH form suggests that predictive asymmetry is of second order form, though this effect is statistically insignificant in returns data, whereas, QGARCH-M form yields no

evidence of a statistically significant effect of volatility on returns at either hourly or quarter-hourly frequency and therefore, no evidence of any volatility feedback through the interaction of predictive asymmetry and risk premium is found. Floros (2008) studied volatility using GARCH models in Egypt and Israel markets and found that the coefficient of the lagged squared return is positive and statistically significant for most specifications and witnessed a strong GARCH effect for both financial markets. The coefficient of lagged conditional variance is significantly positive and less than one, indicating that the impact of old news on volatility is significant and magnitude of the coefficient β is especially high for TASE-100 index, indicating memory in the variance. Mean equation of GARCH-M model, denoted by β_2 is positive but insignificant for both indices, suggesting that higher market-wide risk, proxied by the conditional variance, will not necessarily lead to higher returns. EGARCH models show a negative and significant γ parameter for both indices, indicating the existence of the leverage effect in returns during sample period.

Methodology and Research design

The methodology under this study is divided into the following areas.

1. Data and samples
2. Hypothesis
3. Tools and techniques

Data and Samples

Close level data for this study are collected for ten emerging economies including India. However South Africa and Turkey are not included for the lack of availability of data. All the main indices are included for all the ten countries and the data are collected from yahoo finance website i.e. *in.finance.yahoo.com*. The details regarding the data are incorporated in Table 2.

The closing level data collected are converted into continuously compounded return by applying the following formula

Table 2: Data and Samples

Country	Name of Stock Exchange	Name of the Index	Period Of Study	No of Observation	Data Type
India	Bombay Stock Exchange	BSE Sensex	01.04.2003-29.08.2014	2904	Closing Level Data
China	Shanghai Stock Exchange	SSEC- Shanghai Composite	01.04.2003-29.08.2014	2817	Closing Level Data
Indonesia	Indonesia Stock Exchange (Bursa Efek Indonesia)	JKSE- Jakarta Composite	01.04.2003-29.08.2014	2841	Closing Level Data
Sri Lanka	Colombo Stock Exchange	All share price index (ASPI)	01.04.2003-29.08.2014	2810	Closing Level Data
Pakistan	Karachi Stock Exchange	KSE 100	01.04.2003-29.08.2014	2837	Closing Level Data
Russia	Moscow Exchange	RTS Index	01.04.2003-29.08.2014	2876	Closing Level Data
Brazil	Brazil Stock Exchange (IBOVESPA)	IBOVESPA	01.04.2003-29.08.2014	2861	Closing Level Data
South Korea	Korea Stock Exchange	KOSPI	01.04.2003-29.08.2014	2889	Closing Level Data
Mexico	Mexican Stock Exchange (Bolsa Mexicana de Valores)	IPC	01.04.2003-29.08.2014	2837	Closing Level Data
Hong Kong	Hong Kong Stock Exchange	HSI - Hang Seng Index	01.04.2003-29.08.2014	2867	Closing Level Data

$$r_t = \ln(P_t / P_{t-1}) * 100 \quad (1)$$

Where r_t = logarithmic index return; \ln = natural logarithm;
 P_t = current closing price; P_{t-1} = previous closing price

Hypothesis Testing

The following null hypotheses are being formulated for this study:

H₀: The return data of all the indices are normally distributed

H₀: Volatilities in BRIC countries are more in comparison to developing nations.

H₀: Recent information has more impact on the volatility than the old news

H₀: Asymmetric information impacts more on the volatility of Emerging markets

Tools and Techniques

Normality Test

The data distribution is said to be normal if its skewness is zero and kurtosis is three. The descriptive statistics like mean, standard deviation skewness, and kurtosis of the return data over the period study are calculated by using a statistical software package called E views-7.1. The normality test of the descriptive statistics is carried on by using an asymptotic Jarque-Bera (1981) test statistic. The formula of Jarque-Bera (JB) statistics is stated below:

$$JB \text{ Statistics} = T \left(\frac{S^2}{6} + \frac{(k-3)^2}{24} \right) \quad (2)$$

where T = No. of observations

S = Skewness coefficient

K = Kurtosis coefficient

JB test of normality is the test of the joint null hypothesis if S & K are '0' and 3, respectively.

Stationarity Test

The financial time series data is called stationary if its mean, variance, and auto-covariance at different lags are

same and so time independent. For a stationary series, stocks to the system die away gradually. If the effect of the stocks to the system persists for a longer period, the system will be explosive due to the stock. If the data would not be stationary, no study can be done as non-stationary data lead to spurious regression. The study has therefore conducted stationarity test on the data by using the Augmented Dickey Fuller (1976) test which is stated below:

$$ADF \text{ test statistics } \Delta Y_t = \beta_1 + \beta_2 t + \delta Y_{t-1} + \sum_{i=1}^m \alpha_i \Delta Y_{t-i} + \epsilon_t \quad (3)$$

Heteroscedasticity Test

Heteroscedasticity refers to the unequal variance (σ_t^2) in the error term (u_t) obtained from the regression of Y_t with Y_{t-1} under Ordinary Least Square (OLS) method. In other words, if the coefficient of Y_{t-1} is statistically significant, it indicates the presence of autocorrelation in the return series between Y_t and Y_{t-1} . In the presence of heteroscedasticity, if Classical Linear Regression Method is applied, the best Linear Unbiased Estimates (BLUE) will not be obtained. Hence, the study here intends to develop volatility models under the presence of heteroscedasticity.

In order to know the number of autoregressive (AR), moving average (MA), and ARMA terms, the data of Sensex and Nifty have been tested for period of study by using Box and Jenkins (1976) methodology and Ljung-Box (1978) test.

Box and Jenkins (1976) Methodology

Box-Jenkins (1976) methodology involves three steps:

1. Identification of AR/MA/ ARMA and ARIMA order by correlogram and partial correlogram.
2. Estimation of the parameters (coefficients) of the AR/MA/ARMA and ARIMA model
3. Diagnostic checking of the selected AR/MA/ ARMA/ARIMA model to see that the model selected fits the data reasonably well.

Under this methodology, the study has calculated autocorrelation (AC) and partial autocorrelation (PAC) at various lags from 1 to 30 of the u_t obtained from the OLS of Y_t and Y_{t-1} .

$$\text{AC at } k \text{ lag } (\rho_k) = \text{Cov}u_t, u_{t-k} / \sqrt{\text{Var}u_t \times \text{Var}u_{t-k}} \quad (4)$$

$$\text{PAC at lag } 1 = \text{Ac at lag } 1 = (\rho_{11} = \rho_1^2) \quad (5)$$

$$\text{PAC at lag } 2 (\rho_{22}) = \frac{(\rho_2 - \rho_1^2)}{(1 - \rho_1^2)} \quad (6)$$

The ACs and PACs obtained above are plotted against different lags graphically and the graph so obtained is called correlogram. From this type of visual inspection of a correlogram tentative AR/MA/ARMA model could be determined.

Then as per the Box-Jenkin's methodology, in order to precisely determine the AR/MA/ARMA terms, OLS regression is run on Y_t with Y_{t-1} and u_{t-1} and parameters are noted down. In order to know the significance of regression coefficients i.e. regression coefficients of lagged term, Akaike's Information Criteria (AIC) and Schwarz's Bayesian Information Criteria (SBIC) are applied. The formula used for AIC and SBIC are stated below:

$$\text{AIC Value} = T \left[l_n \left(\sum_{t=1}^T u_t^2 \right) \right] + 2k \quad (7)$$

$$\text{SBIC Value} = T \left[l_n \left(\sum_{t=1}^T u_t^2 \right) \right] + k \ln(T) \quad (8)$$

where,

K = No. of parameters to be estimated

T = total No. of Observations

ln = Natural logarithm

Ljung-Box (1978) test

The significance test of the values of (AC) and (PAC) are done by the Q-Statistics developed by Ljung-Box (1978). The formula for Q-Statistics is given below:

$$Q_m = T(T+2) \sum_{i=1}^m \frac{\rho_i^2}{m-1} \approx x_m^2 \quad (9)$$

Q_m = Ljung-Box Q statistics

T = No. of observations

i = No. of lags varies from 1, 2,, m.

ρ_m^2 = Sample ac at lag m

x_m^2 = chi-square distribution 'm' degrees of freedom.

Volatility Modeling

The study has taken care of calculating the volatility of all the indices over the period of study by recognising the characteristics of the stock market data like heteroscedasticity, clustering, asymmetry and persistence. The study by using AR and MA terms under Box-Jenkins (1976) and Ljung-Box (1978) has developed the ARMS model for the calculation of volatility of Sensex and Nifty.

The Auto Regressive Conditional Heteroscedasticity (ARCH) developed by Engle (1982) has been used to estimate the conditional variance under the presence of heteroscedasticity. Bollerslev (1986) developed a model which considers a combination of squared residuals at lag 'q' and conditional variance at lag 'p' and the model is named as Generalised ARCH or GARCH (q, p) model. There are several extensions of GARCH models which calculate conditional variance in the presence of asymmetric nature of shocks and persistence features of the shocks to the market.

To capture the asymmetric effect of news, Nelson (1991) proposed EGARCH model for the estimation of conditional volatility and hence used in this study to capture the asymmetric volatility of all the indices.

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \alpha_2 u_{t-2}^2 + \dots + \alpha_q u_{t-q}^2 \dots \dots \dots \text{ARCH (q)} \quad (10)$$

where α_0 = is the measure of long term constant volatility i.e. unconditional variance estimation.

$\alpha_1 \dots \alpha_q$ are the coefficients of the residuals / error terms

α_1 = is the measure of persistence – reflects the tendency of the volatility of share or index being affected by previous day.

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta \sigma_{t-1}^2 \text{ GARCH (1,1)} \quad (11)$$

where α_0 , α_1 and β are the coefficients of the regression

α_0 is the measure of long term constant volatility i.e. unconditional variance estimation.

$\alpha_1 + \beta$ represents persistence- tendency of an index being affected by the previous days' volatility

α_1 is the coefficient of the squared error term of the previous day, describes volatility due to one day old news.

β is the coefficient of the lagged conditional variance

and describes the volatility due to news which are old by more than one day.

$$\ln(\sigma_t^2) = \alpha_0 + \alpha_1 \left[\frac{|u_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{2}{\pi}} \right] + \gamma \frac{u_{t-1}}{\sqrt{\sigma_{t-1}^2}} + \beta \ln(\sigma_{t-1}^2) \quad (12)$$

EGARCH(1,1)

1. The equation for the conditional variance is in log-linear form. Regardless of magnitude of $\ln(\sigma_t^2)$, the implied value of σ_t^2 can never be negative. Hence, it is permissible for the coefficients to be negative.
2. Instead of using the value of u_{t-1}^2 , EGARCH model uses the level of standardized value of u_{t-1} [i.e.

$\frac{u_{t-1}}{\sqrt{\sigma_{t-1}^2}}$]. Nelson argues that this standardisation

allows for a more natural interpretation of the size and persistence of shocks. After all, the standardised value of u_{t-1} is a unit-free measure.

3. The EGARCH model allows for leverage effects.

If $\frac{u_{t-1}}{\sqrt{\sigma_{t-1}^2}}$ is positive, the effect of the shock on the

log of the conditional variance is $\alpha_1 + \gamma_1$. If $\frac{u_{t-1}}{\sqrt{\sigma_{t-1}^2}}$

is negative, the effect of the shock on the log of the conditional variance is $-\alpha_1 + \gamma_1$.

Table 3: Descriptive Statistics of India

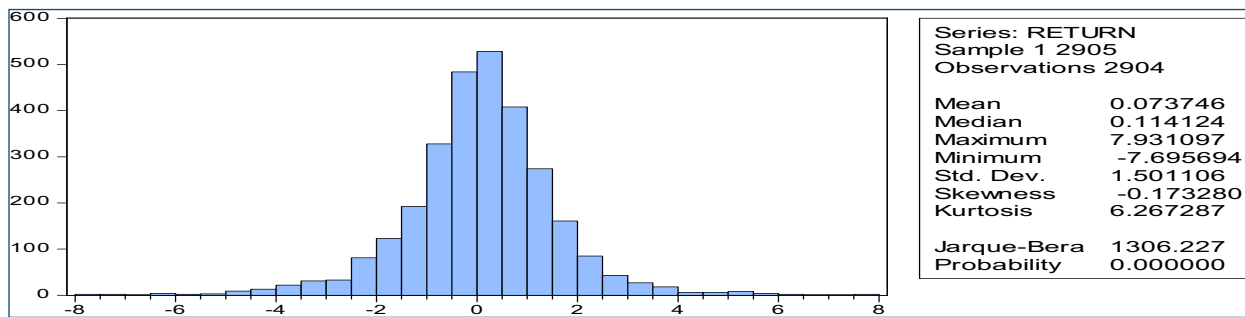


Table 4: Descriptive Statistics of Brazil

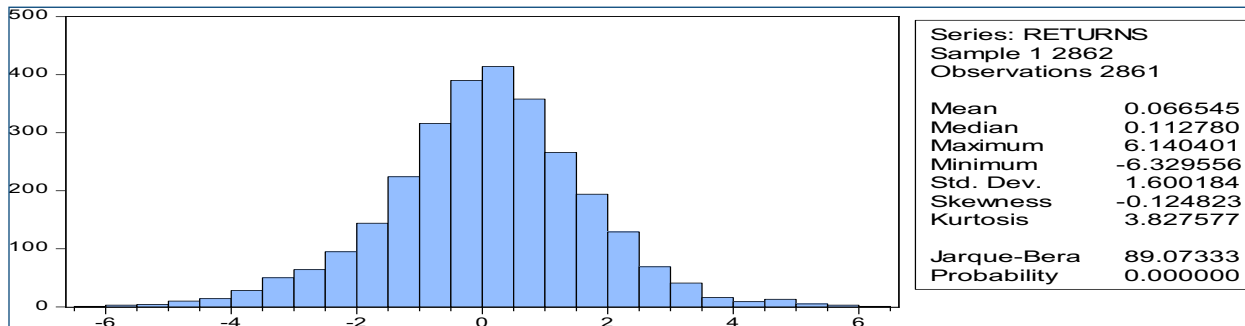


Table 5: Descriptive Statistics of Srilanka

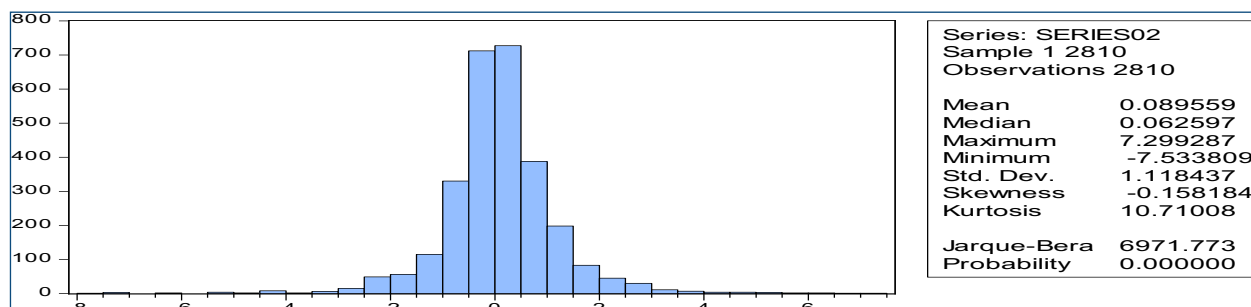
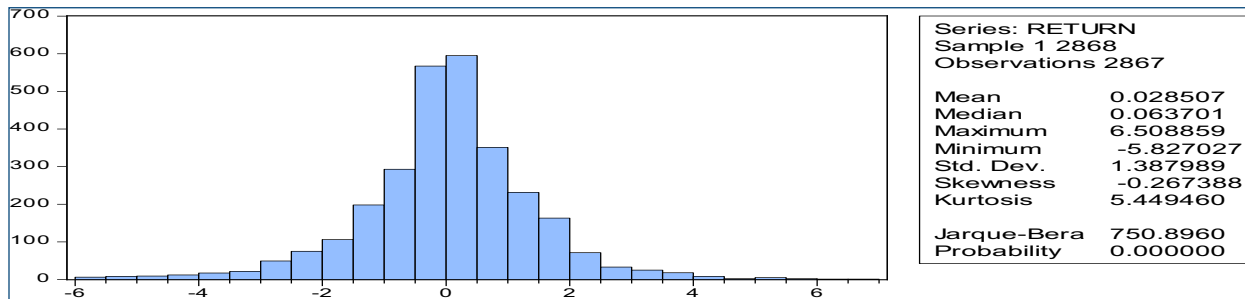
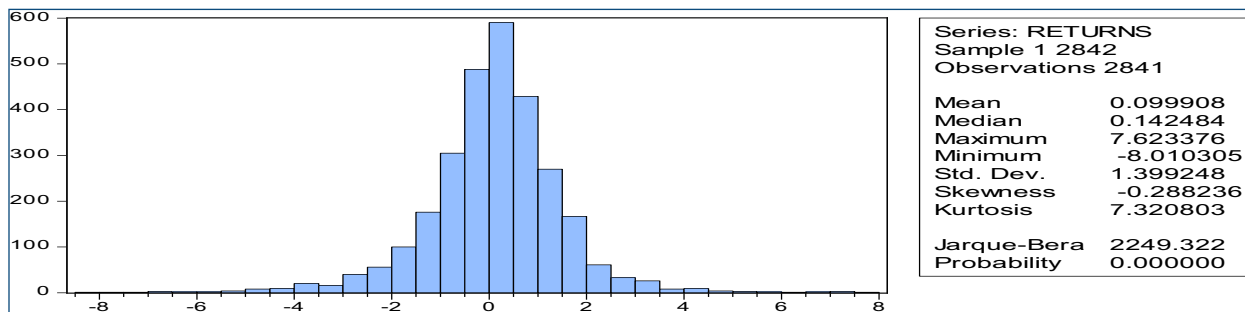
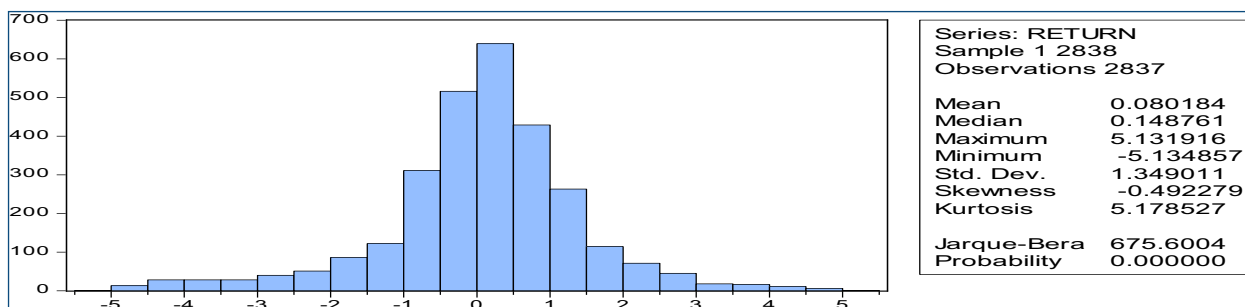
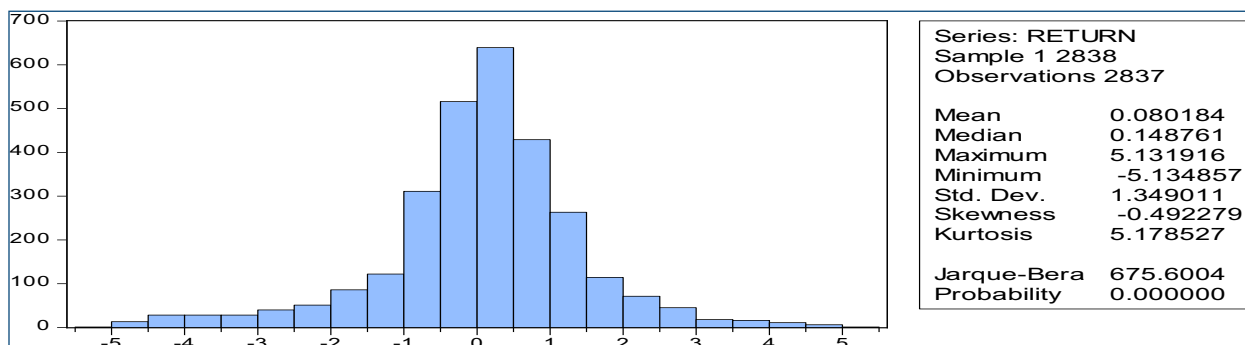
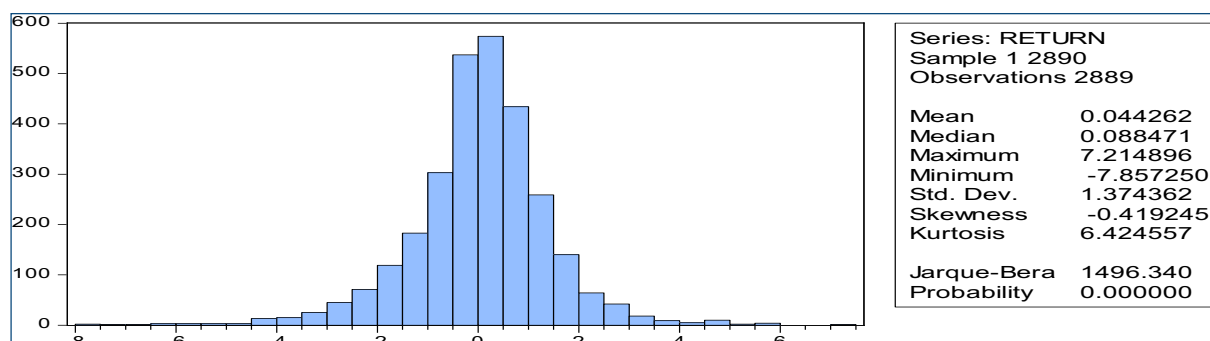
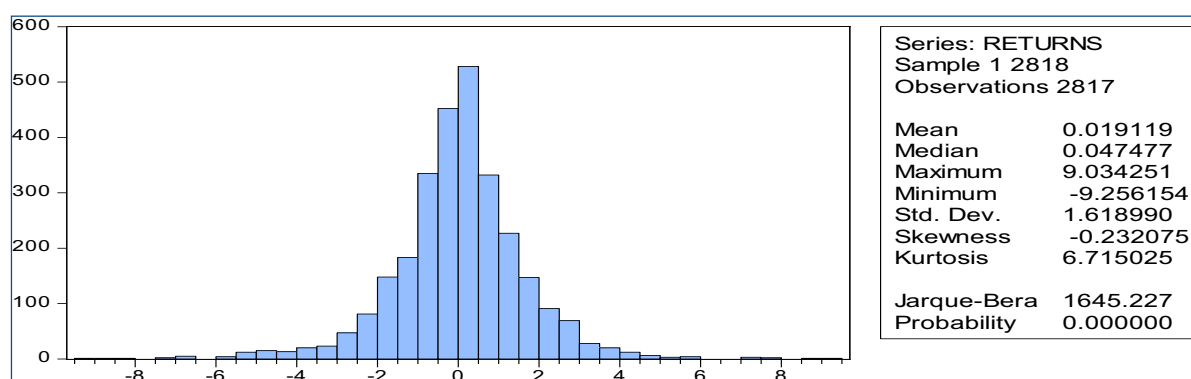
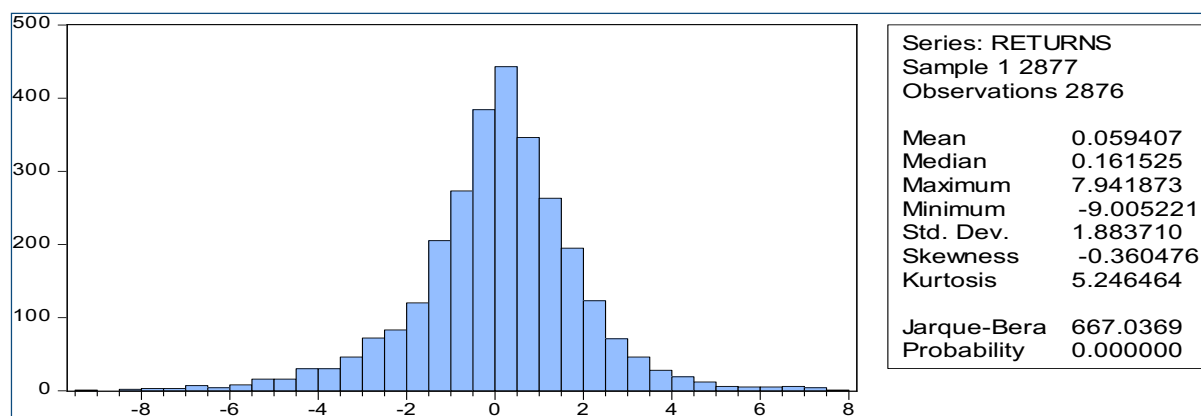


Table 6: Descriptive Statistics of HongKong**Table 7: Descriptive Statistics of Indonesia****Table 8: Descriptive Statistics of Pakitan****Table 9: Descriptive Statistics of Mexico****Table 10: Descriptive Statistics of South Korea**

**Table 11: Descriptive Statistics of China****Table 12: Descriptive Statistics of Russia**

Analysis of Results

Analysis Table 3 through 12

The log returns of the indices are not normally distributed as the skewness is not equal to zero and kurtosis is more than 3. If the kurtosis is greater than 3, it suggests leptokurtic pattern (slim or long tailed) of the indices. As the skewness in most of the indices are less than zero or negative, it suggests the indices are skewed towards the

left. The Jarque Bera Test statistics follows Chi Square distribution, with two degrees of freedom, and the null hypotheses that the log returns of the indices are normally distributed can be rejected, as the p values are also significant.

Table 13 represents unit root calculations and suggests that the calculated values of ADF test statistics for all the indices under the consideration are more than the table value at 1%, 5% and 10% level of significance thus indicate that the continuously compounded returns of all the stock markets are stationary.

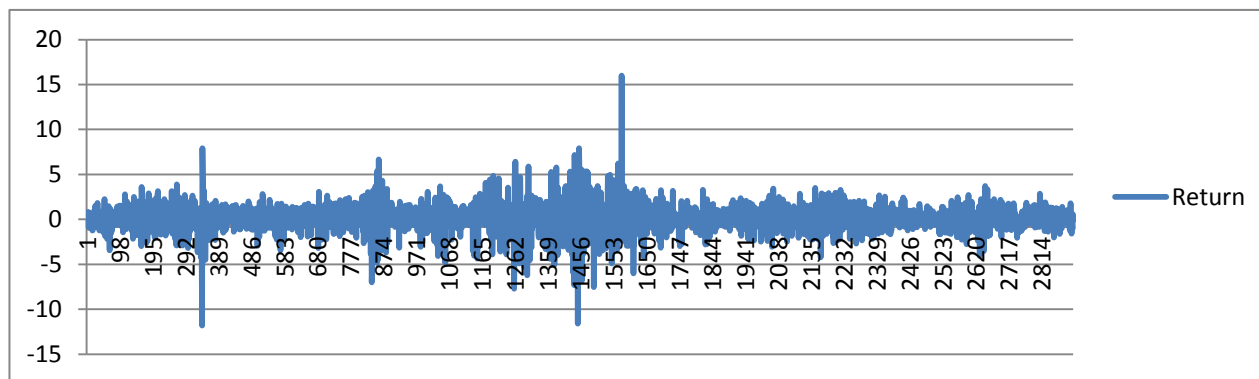
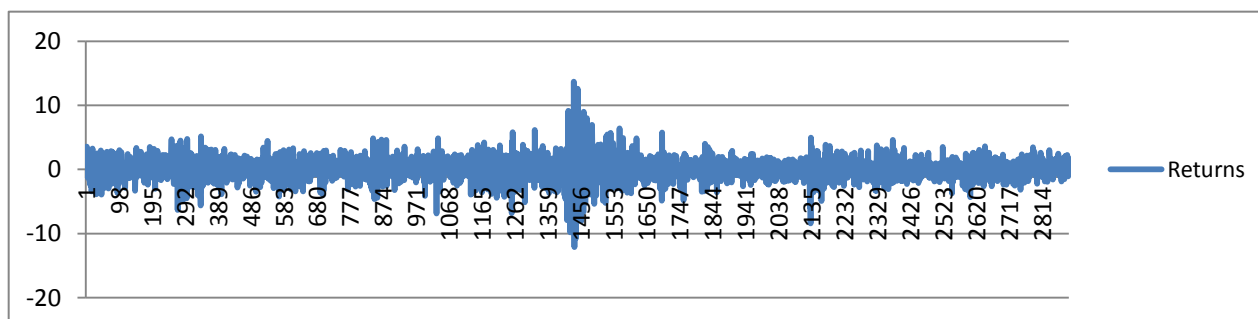
Table 13: Unit Root/ Test of Stationarity

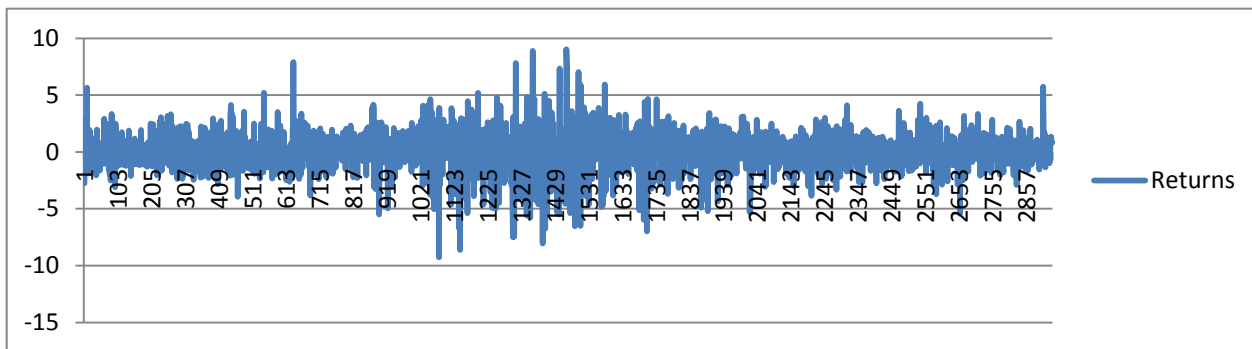
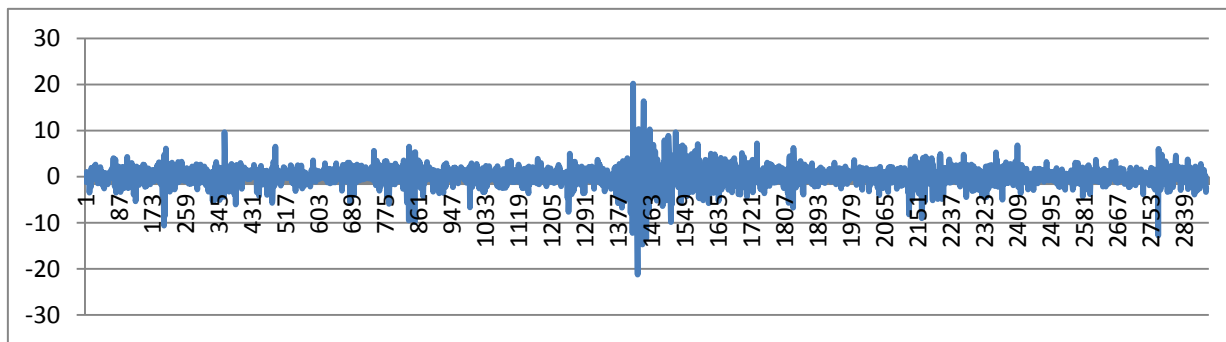
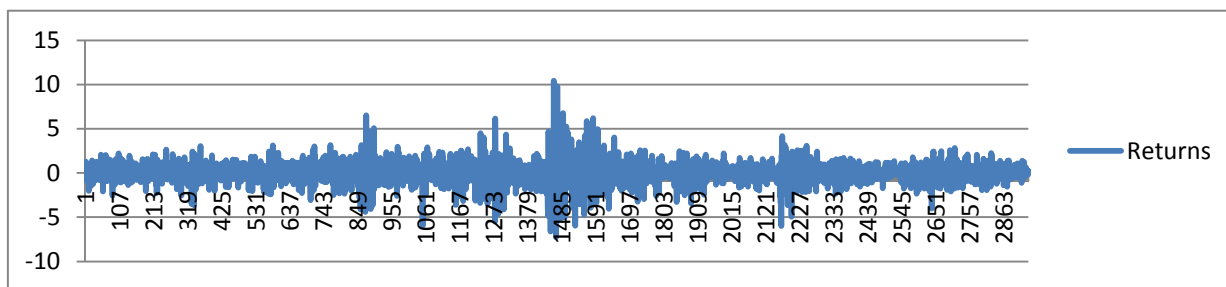
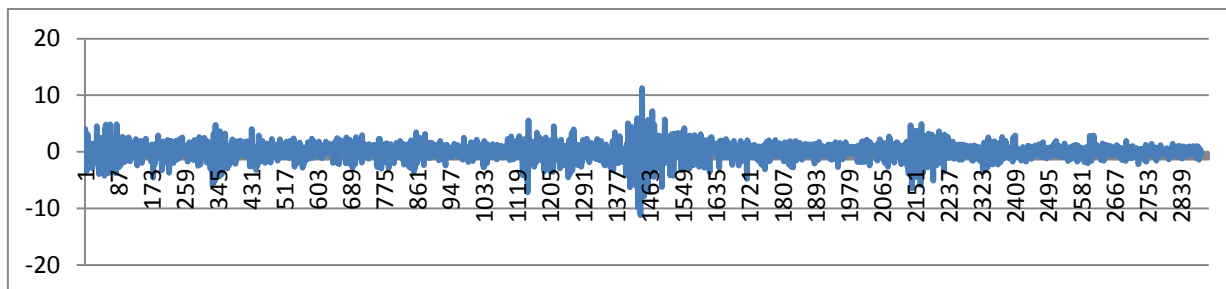
	India	Brazil	Srilanka	HongKong	Indonesia
Unit Root Test (ADF Test)	-50.87	-53.09	-43.77	-52.75	-47.76
1	-3.959402	-3.959591	-3.960327	-3.959373	-3.960318
5%	-3.410473	-3.410565	-3.410926	-3.410458	-3.410921
10%	-3.127001	-3.127055	-3.127269	-3.126992	-3.127267
Critical Value	0.000	0.000	0.000	0.000	0.000
Probability Values					
	Pakistan	Mexico	South Korea	China	Russia
Unit Root Test (ADF Test)	-46.42	-49.93	-53.82	-52.74	-48.60
1%	-3.960006	-3.959830	-3.959119	-3.959694	-3.959402
5%	-3.410768	-3.410682	-3.410334	-3.410615	-3.410473
10%	-3.127176	-3.127125	-3.126918	-3.127085	-3.127001
Critical values	0.000	0.000	0.000	0.000	0.000
Probability Values					

Test of Volatility

Figs.1 through 10 suggest the volatility graphs of the indices considered for this study and represented in the

form of continuously compounded return over a period of ten year. Volatility graphs suggest outliers for all indices particularly during the period of global economic crisis witnessed mostly in 2008. Return series of some indices

**Fig. 1: SENSEX Return Data-India****Fig. 2: IBOVESPA Return Data-Brazil**

**Fig. 3: SSEI Return Data- China****Fig. 4: RTSI Return Data-Russia****Fig. 5: IPC Return Data- Mexico****Fig. 6: KOSPI Return Data-South Korea**

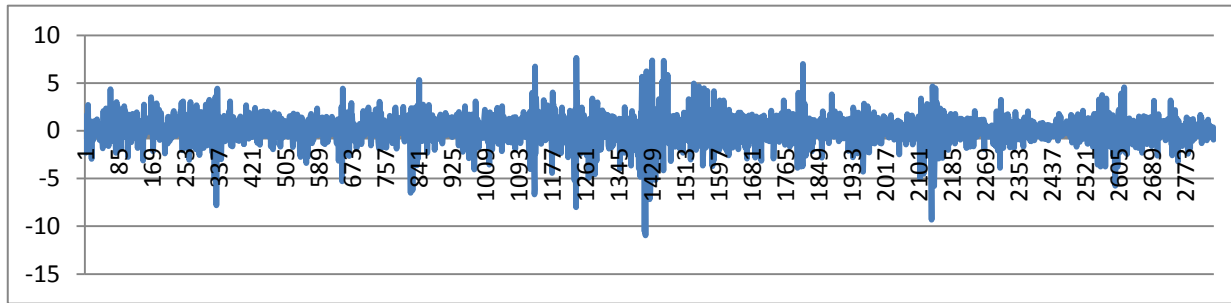


Fig. 7: JSKE Return Data- Indonesia

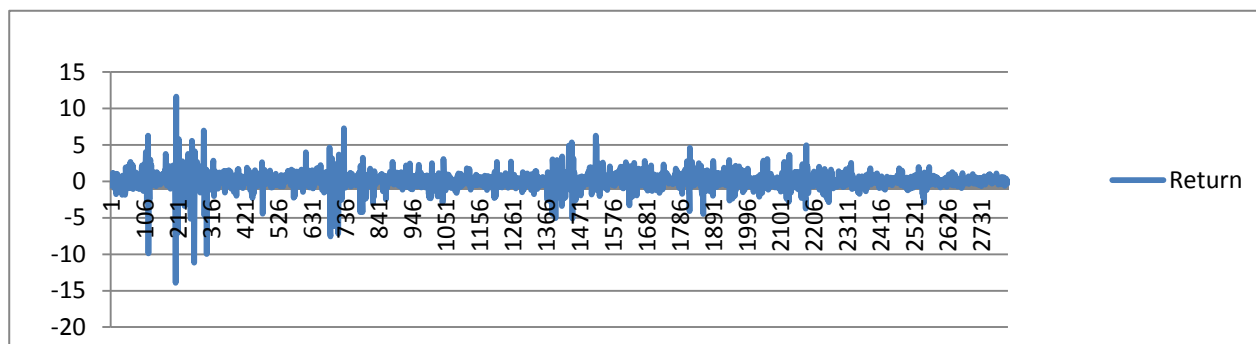


Fig. 8: ASPI Return Data- Srilanka

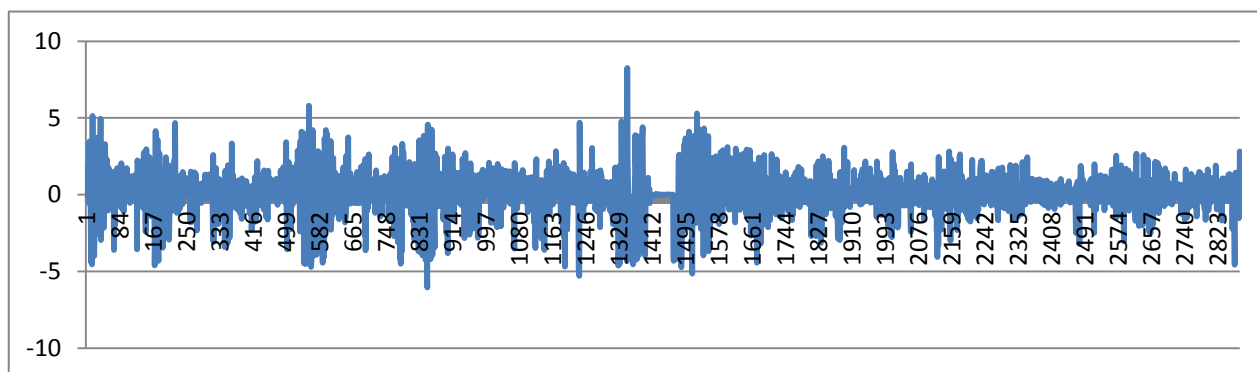


Fig. 9: KSE 100 Return Data-Karachi Pakistan

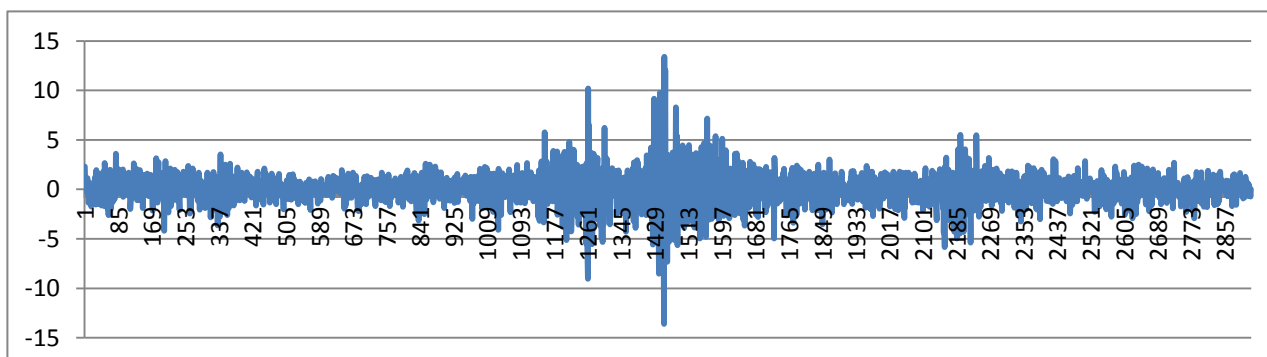


Fig. 10: HSI Return Data- Hong Kong

Table 14: Volatility by the measure Standard Deviation

Volatility (%)	India	Brazil	Sri-lanka	Hong Kong	Indo-nesia	Pakistan	Mexico	South Korea	China	Russia
Standard Deviation	15	16	11	13.87	14	13.49	13.49	13.74	16	18.83

also occurred to be zero during some periods which are subsequently removed to filter the data along with the outliers for estimating volatility. Volatility graphs also suggest a huge volatility during the year of economic crisis for all indices undertaken for this study. Certain amount of volatility also visible for India during the initial period under the study (2003-04) and a strong volatility is seen during the 19th of May 2009 because of declaration of election results at Centre. For Srilanka, volatility is also visible during the year 2003-04 and Indonesia return data found to be more spiked over the period of study. Hong Kong return data are found to be cluster over the period of study except for the period of economic crisis. SSEI, China return data also suggest more volatility in the form spiked.

Table 13 suggests that calculated volatilities by the measure of standard deviation in the BRIC countries are relatively higher than the other emerging economics and hence it can be interpreted that BRIC stock exchanges are happened to be more volatile during the period of study in comparison to other emerging economics.

Table 14 represents appropriate ARMA models, fitted by employing BOX-Jenkins methodology, for all the indices under the study. Fitted ARMA models mostly follow moving average pattern except for Srilanka, Hong Kong and Pakistan where Autogressive pattern is found with the moving average pattern, however for Hong Kong return series suggest a pure AR pattern. Longer moving average patterns in the return series suggest persistent shocks in the system over a period of time.

Table 15: Fitted Autoregressive Moving Average (ARMA) Model

Model	India	Brazil	Sri-lanka	Hong Kong	Indo-nesia	Pakistan	Mexico	South Korea	China	Russia
ARMA	MA (1) MA (11) MA (17)	MA (2)	AR (1) MA (4) MA (8) MA (10) MA (11)	AR (2)	MA (1)	AR (1) MA (3) MA (4) MA (9)	MA (1) MA (6)	MA (4) MA (8) MA (14) MA (15)	MA (3) MA (4) MA (6) MA (15)	MA (1) MA (5) MA (13)

Table 16: Volatility Estimation Employing GARCH and EGARCH

India	Model	Model
	GARCH	EGARCH
	α_0	0.033379
	α_1	0.094767
	β_1	0.88976
	$\alpha_0 + \alpha_1 + \beta_1$	1.00
Brazil	GARCH	
	α_0	0.040611
	α_1	0.049916
	β_1	0.933752
	$\alpha_0 + \alpha_1 + \beta_1$	1.00
China	GARCH	
	α_0	0.025262
	α_1	0.045459
	β_1	0.943775

India	Model		Model	
	$\alpha_0 + \alpha_1 + \beta_1$	1.00	β_1	0.988667
Russia	GARCH			
	α_0	0.098524	α_0	-0.07701
	α_1	0.083721	α_1	0.161196
	β_1	0.88692	γ_1	-0.04974
	$\alpha_0 + \alpha_1 + \beta_1$	1.00	β_1	0.960792
South Korea	GARCH			
	α_0	0.014052	α_0	-0.10871
	α_1	0.071525	α_1	0.154035
	β_1	0.920941	γ_1	-0.08875
	$\alpha_0 + \alpha_1 + \beta_1$	1.00	β_1	0.978327
Mexico	GARCH			
	α_0	0.018446	α_0	-0.10546
	α_1	0.078074	α_1	0.147604
	β_1	0.910024	γ_1	-0.09249
	$\alpha_0 + \alpha_1 + \beta_1$	1.00	β_1	0.97836
Hong Kong	GARCH			
	α_0	0.00957	α_0	-0.07874
	α_1	0.048915	α_1	0.110451
	β_1	0.945203	γ_1	-0.04709
	$\alpha_0 + \alpha_1 + \beta_1$	1.00	β_1	0.986849
Srilanka	GARCH			
	α_0	0.051742	α_0	-0.33506
	α_1	0.265401	α_1	0.444684
	β_1	0.721274	γ_1	-0.04339
	$\alpha_0 + \alpha_1 + \beta_1$	1.00	β_1	0.932752
Indonesia	GARCH			
	α_0	0.060962	α_0	-0.14987
	α_1	0.125276	α_1	0.233294
	β_1	0.845768	γ_1	-0.08485
	$\alpha_0 + \alpha_1 + \beta_1$	1.00	β_1	0.953201
Pakistan	GARCH			
	α_0	0.077503	α_0	-0.16405
	α_1	0.138171	α_1	0.280939
	β_1	0.81608	γ_1	-0.12462
	$\alpha_0 + \alpha_1 + \beta_1$	1.00	β_1	0.895392

Table 15 represents the estimated volatility by employing GARCH(1,1) and EGARCH(1,1) for all the emerging markets considered for this study. The table suggests that the coefficients α_0 , α_1 , β_1 and γ_1 are statistically significant and are within parametric restriction for all the period under the study, thus implying a greater impact of shocks (or news) on volatility. A significant

ARCH coefficient α_1 indicates a large shocks on day $t-1$ leads to large (conditional) variance on day t . α is the “news” component that explains that recent news has a greater impact on price changes and it implies the impact of yesterday’s news on today’s volatility. The GARCH coefficient β_1 measures the impact of “old news”. In this study EGARCH model is used to capture the tendency for

negative shocks to be associated with increased volatility. The logarithm form of the conditional variance implies that the leverage effect is exponential (so the variance is non-negative) is also shown in the table. The presence of leverage effects can be tested by the hypothesis that $\gamma < 0$ and if $\gamma \neq 0$, then the impact is asymmetric. α_1 for Srilanka, Indonesia and Pakistan are found to be the highest in comparison to other markets. β_1 for most of the countries is found to be high except for Srilanka. Presence of asymmetry in volatility of the indices during the periods suggest that the volatility is more in the markets whenever negative information flows into the market in comparison to the positive news. Pakistan has the highest value of γ_1 which suggest more impact of market volatility because of negative information.

Conclusion

Volatility estimations across the countries are found to be high during the period of global economic crisis. Asymmetric information found to be impacted all the countries during the period of study and volatility in Pakistan, Srilanka, and Indonesia are mostly impacted through recent information.

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Forecasting Volatility Spillover of Information Technology Sector Stocks in India: An Application of ARMA & GARCH Model

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Abstract

Information Technology (IT) is a predominant sector in market over last two decades due to high interdependence of other sectors over this discipline. Investment in this sector is growing with advent of emerging technologies. The research paper aims to portray the real picture in front of investors using forecasting the share prices of Indian IT sector. Considering the importance of the specific sector, this study revolves around forecasting the share prices and volatility of IT sector stocks. Three large cap stocks from the S&P BSE Index are considered based on their market capitalisation at the time of selection. Auto regressive moving average (ARMA) is applied to forecast its prices while Generalised Auto Regressive Conditional Heteroscedasticity (GARCH) is use to forecast volatility. The research has been conducted for 3 major IT service companies named Tata Consultancy Service (TCS); Infosys & Tech Mahindra with Monthly data taken of 12 years. It is found that the forecasted prices of stocks of IT sector are showing an increasing trend and the forecasted volatility associated with these stocks are high as compared to other sectors. The volatility is highest in Tech Mahindra while least in TCS.

JEL Codes: G31, G32

Keywords: Forecasting, Information Technology, Arma, Gaarch, Stock Price

Introduction

In last decade, InfoTech Industry has become an international and globalised sector. In information technology sector, due to rapid increase in research & development in manufacturing techniques for the growth of computer hardware and innovative and advent practices

in software has raise the demand in underdeveloped as well as developing countries.

The Indian stock market is predominant by three sectors namely, service, manufacturing, and agriculture sector. The most amazing thing about IT sector is that it cannot be avoided, every company having high dependence on technology for its betterment. India is in its expansion stage, so it's an opportunity to dive into the future. It's very important to understand the ongoing trends, forecast, depth and gravity of the situation in a company from investment point of view. The investment in this sector is also increasing exponentially due to more reliability and resilience by this sector. Still there is some uncertainty is associated with investment decision due to some obsolete and neglecting practices or vague idea about share prices of this sector. The sector demands huge investments, huge risks (definitely not for the faint hearts) but surely as they say 'To get higher returns on investment, one has to take higher risk'.

India is one of the fastest growing IT services markets in the world. It is the world's largest sourcing destination, accounting for approximately 52% of the 124-130 billion market. India has the potential to build a US\$ 100 billion software product industry by 2025, according to Indian Software Product Industry Roundtable (iSPIRIT). The contribution of the IT sector to India's GDP rises too, approximately to 8 % (2015) from 1.2% (1998) and due to factors as emerging technologies, manpower. Indian IT companies are expected to grow at a CAGR of 13.2-15.2% over the next five years.

The industry grew at a CAGR of 13.1% during FY08-13. Total export from the IT-BPM sector were estimated at US\$ 76 billion during FY013. BPM sector accounted for 3.5% of total IT exports. The IT outsourcing sector

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is expected to see exports growing by 1-15% during FY15. The sector ranks 4th in India's total FDI share and accounts for 7% of total Private Equity and Venture Investments in the country (Nasscom).

The previous research mainly highlights the factors affecting the future prices. The main problem in this area lies with the investor in the prediction of future prices of stocks and to take informed decisions. There has been no proper study on the IT sector for longer duration and the investors, fund managers, and business class is unaware of the risk in terms of volatility and return in IT sector. Therefore forecasting of the returns of the sector is needed as well as the volatility of the sector needs to be estimated.

Poon and Granger (2012) explained volatility forecasting is an important task in financial markets, by which it has grabbed the attentions of academics and practitioners over last two decades. Volatility is not same as risk but when it is stated as uncertainty it becomes a key input to many portfolio creations and investment decisions. A good forecast of the volatility of prices over investment is a good start for the assessment of risk.

In finance volatility sometimes referred to as standard deviation or variance. Diebold (1998) mentioned forecasting will differ depending on different factors like current level of volatility, forecast horizon and volatility structure but volatility structure should be same.

Objectives

The objectives of this paper revolve around majorly forecasting share prices and volatility,

1. To study the current share price of IT companies in India.
2. To forecast the share price of IT companies using ARMA model.
3. To check the volatility in share prices of IT companies through EGARCH

Literature Review

Various researchers highlighted the significance of volatility spillover and forecast of share prices from time to time in numerous sectors with different approaches as well. Edwards, Biscarri, and Gracia (2003) analyses the behaviour of stock prices in six emerging countries

and conclude that after financial liberalisation Latin American markets are more stable in comparison to Asian economies, especially Korea which is in their recovery stage. DeBondt and Thaler (1985) concluded with data past returns inform expected returns. Past returns are positively related to future average returns, which are influenced by the microstructure and data snooping biases. Kaminsky and Schumkler (2001, 2002) also analysed the behaviour of stock prices over financial cycles that stock markets were steadier when liberalised and volatility increases immediately following liberalisation in 28 countries where financial liberalisation took place. Jagdeesh (1990), Smirlock and Starks (1985) and Brush (1986) technical analysis is based on the rationale and the correlation between price and volume reveals market behaviour, prediction based on the past activity and by analysing the trend.

Volatility Assessment Practices

Multiple methods and techniques implied globally in order to assess share price as well as volatility. Brown (1990), Engle (1993), and Aydermir (1998) applied the risk metrics model uses the EWMA which is the more flexible version of exponential smoothing in which weight depends upon size and sign. Bera and Higgins (1993), Kroner (1992) Engel and Nelson (1994), and Diebold and Jose Lopez (1995) show that the importance of ARCH model does not make use of the sample standard deviation and formulate conditional variance. It is one step ahead forecast but by further study it is found out that GARCH is more parsimonious model than ARCH and GARCH is even famous for most of the financial time series.

According to Agiray, 2012 GARCH always and consistently performs well in all sub-periods and under all evaluation measures. Cumby, Figlewski and Hasbrouck (1993) state that EGARCH is better, whereas Cao and Tsay (1992) conclude that TAR is the best forecast for large stocks and EGARCH gives best forecast for small stocks.

Assimakopoulos and Nikolopoulos (2000) after conducting many surveys on available forecasting techniques for stock market volatility, conduct a forecasting competition with seven different approaches which include random walk model, exponential smoothing model, mean model, and ARCH family model. The result was that on the average, say 1-4 horizons, SES, EGARCH

and Random walk present same accuracy and for short term forecasting, say 2-4 horizons, EGARCH produces the most accurate forecast.

Basin Capital Markets (PACAP) Research Center at the University of Rhode Island (USA) and the SINOFIN Information Service Inc., affiliated with China used ARIMA model to solve real world problem in the stock market by forecasting the stock prices with the top four companies in Nifty midcap 50. ARIMA was also used for predicting price specially electricity prices for next day. This research forms the applicability of ARMA & GARCH model in IT sector which is not applied in selected companies so far for the mentioned time duration.

Research Methodology

The methodology opted for this research is purely secondary in nature. Data are collected from BSE India website, Capitaline database, and online stock market historical data sources. No primary data are used in this research report. The companies representing the IT sector Index is chosen as per their market capitalisation. Three companies, namely Tata Consultancy Services (TCS), Infosys, and Tech Mahindra were chosen based on their highest market capitalisation highlighted in Table 1 which represents the large caps in the market, as well. The data used here are closing prices of three stocks from the date when the companies are listed in stock market.

Forecasting Share Prices (ARMA)

With time many authors have applied Auto Regressive Moving Average (ARMA) model which is the most simple and widely used technique in forecasting the stock prices. Time series analysis forms an important part of the statistics which analyses dataset to study characteristics of the data and helps predicting future values of the series based on the characteristics. The difference between ARMA and ARIMA is that ARIMA converts a non-stationary data to a stationary data before working on it. ARMA model is widely used to predict time series data. Since it is necessary to identify a model for forecasting stock price movement it is better to use ARMA than forecasting directly as it gives more accurate results. AR (1) (Auto Regressive), MA (1) (Moving Average), and ARMA (1, 1) can be described by a series of equations 3.1; 3.2 & 3.3.

$$(AR\ 1): y_{(t)} = a_{(t)} * y_{(t-1)} + e_{(t)} \dots\dots\dots (3.1)$$

where $y_{(t)}$ is the mean-adjusted series in period t , $y_{(t-1)}$ is the series in the previous period value, $a_{(t)}$ is the lag-1 autoregressive coefficient and $e_{(t)}$ is the noise.

The moving average (MA 1) model is given by:

$$Y_{(t)} = e_{(t)} + c_{(1)} * e_{(t-1)} \dots\dots\dots (3.2)$$

where $e_{(t)}$, $e_{(t-1)}$ is a form of ARMA model in which time series is regarded as a moving average (unevenly weighted) and $c_{(1)}$ is the first order moving average coefficient.

The order of ARMA model is included as ARMA (1, 1) where p is the autoregressive order and q is the moving average order is applied taking cumulative effect of equation 3.1 & 3.2 in equation 3.3. The most frequently used ARMA model is

$$Y_{(t)} = d + a_{(t)} * y_{(t-1)} + e_{(t)} + c_{(1)} * e_{(t-1)} \dots\dots\dots (3.3)$$

Forecasting Volatility (GARCH)

Generalised Auto Regressive Conditional Heteroscedasticity (GARCH) first developed by Bollerslev (1986) which is similar in spirit to an ARMA model. In GAARCH model, the GAARCH model can be applied through equation 3.4.

$$\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \dots\dots\dots (3.4)$$

where $\alpha_0 > 0$, $\alpha_1 > 0$, $\alpha_1 + \beta_1 < 1$, so that our next period forecast of variance is a blend of our last period forecast and last period's squared return.

Data Analysis

Forecasting Share Prices Using ARMA & Volatility using GARCH

Analysis of stock price forecast and volatility spillover are two stages applied first using ARMA(1,1) model to forecast stock prices and secondly using GARCH (1,1) for volatility forecast. The forecast is done in selected three companies TCS, Infosys, and Tech Mahindra from IT sector on the basis of their PE Ratio and EPS as mentioned in Table 1.

Table 1: Analysis of companies selected

Companies	PE Ratio	EPS	Market Cap (CR)
TCS	28	89.5	491712.1
Infosys	23.5	83.4	225525
Tech Mahindra	24.1	106.5	60553.94

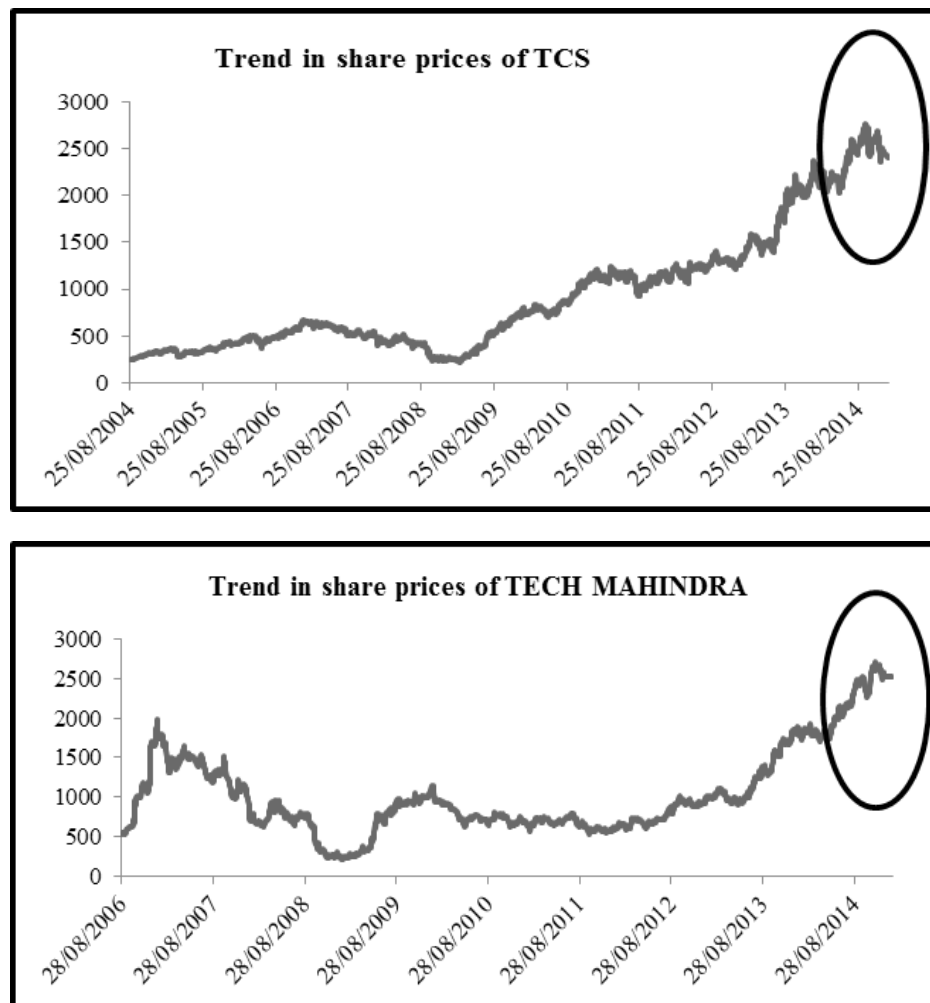
The three forecasting graphs shown in Fig. 1 (a, b & c) share price movements over last few years and the forecasted share price trend for all the three companies stock. IT industry is booming and so its share prices. The industry grew with a CAGR of 13% during FY08-13. Highlighted Fig. 1 shows the forecasted period.

The noise, which is the error, is False for all three stocks. The fitness of stock is 1, which means 100% while the skewness is calculated standard error of skewness which is square root of 6/number of sample and then multiplied by 2. The range should be +ve or -ve after multiplying

by 2. If it is more than range- positively skewed if less negatively skewed and within than normal distributed. Kurtosis is calculated as square root of 24/number of sample then multiplying it by 2. Thus if it is more than range it is Leptokurtic and less than it is Pletokurtic.

In order to find a proper model for the data collected and forecasting of its volatility, auto correlation (ACF) and partial auto correlation (PACF) (Figs. 2 and 3) were done. Similar graphs are being plotted for Infosys and Tech Mahindra.

The graphs show no significant auto correlation in the data so an ARMA model is not justified to take out the volatility of the stocks. This being said the data do possess fat tails and the taking out exponential weighted moving average will be ideal. Therefore the use of E-GARCH from the GARCH family would be ideal in this case (Harrison & Moore, 2012).



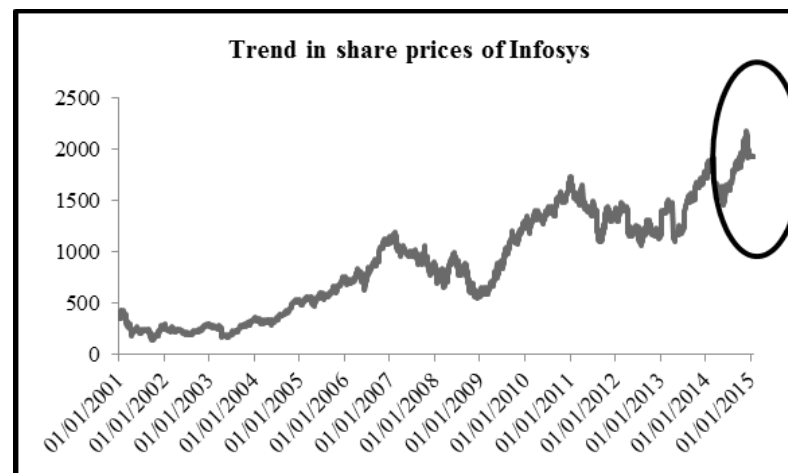
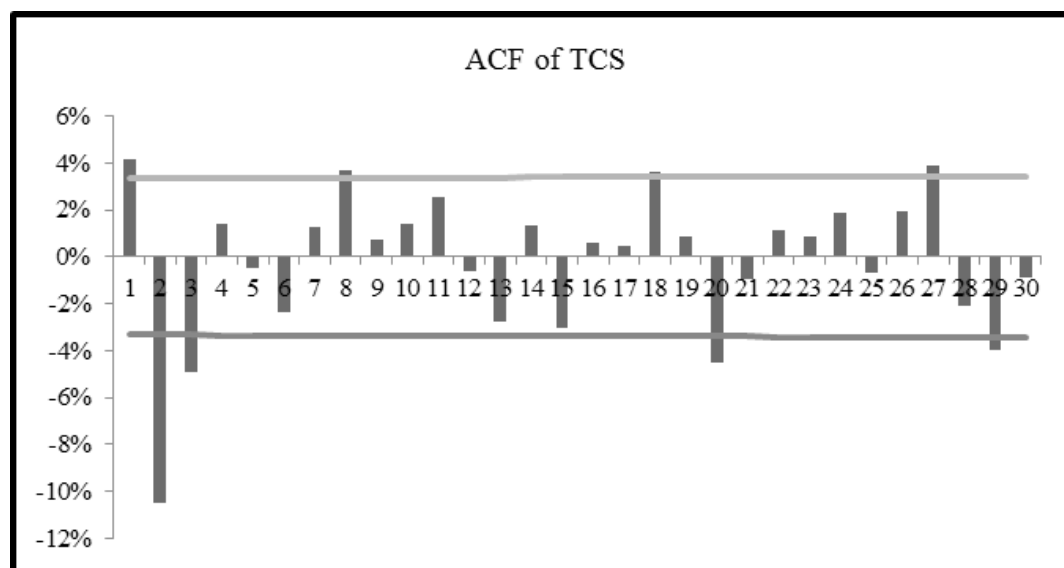
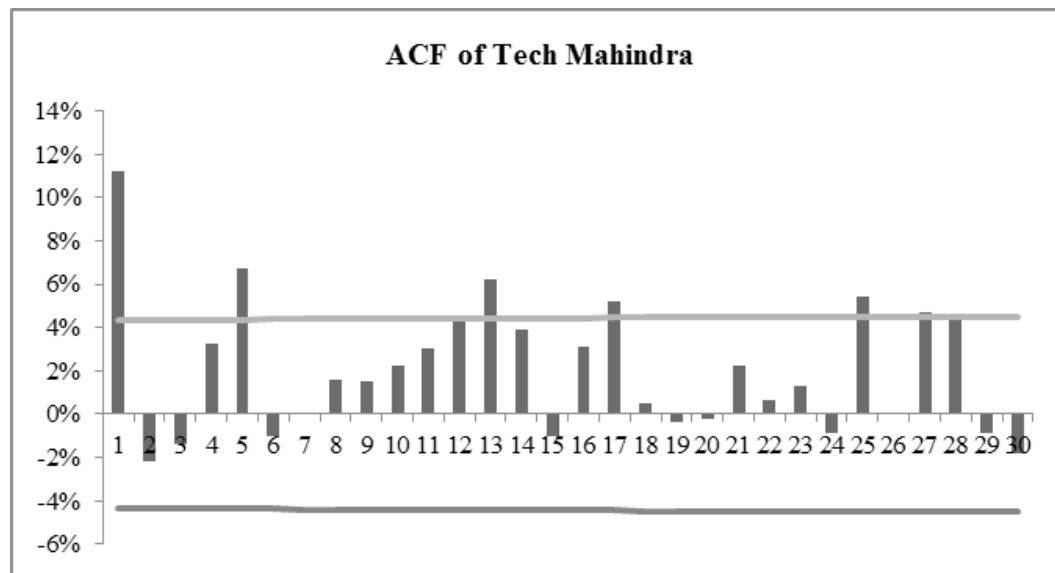


Fig. 1: Share Price Forecasted Data a) TCS, b) Infosys, & c) Tech Mahindra



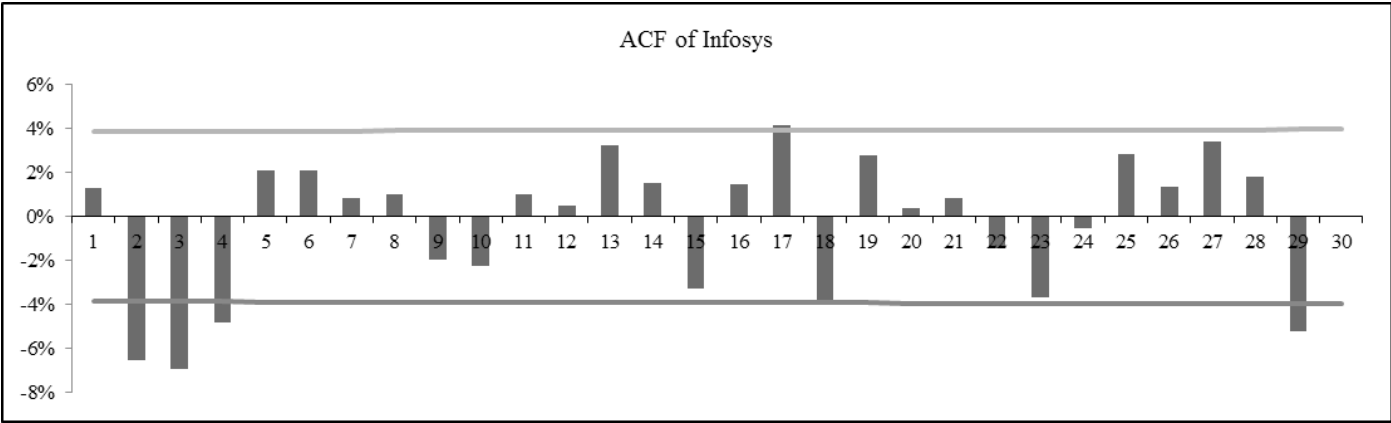
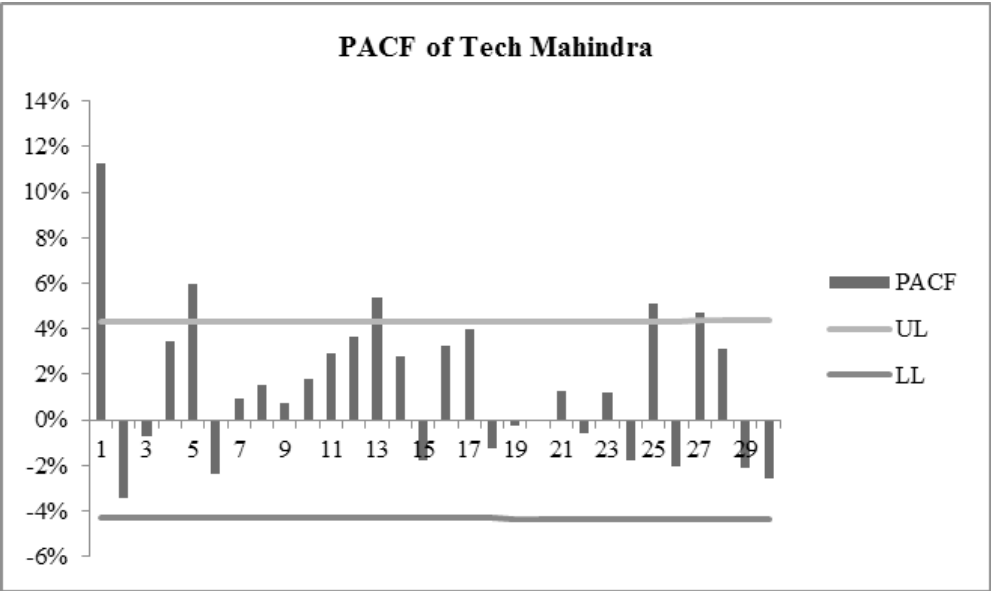
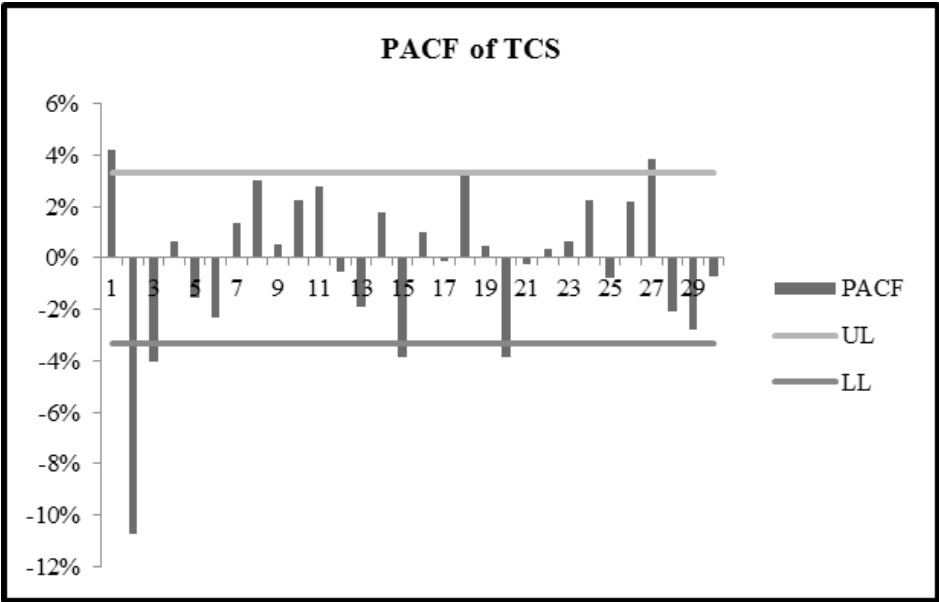


Fig. 2: Auto correlation for a) TCS, b) Tech Mahindra & c) Infosys



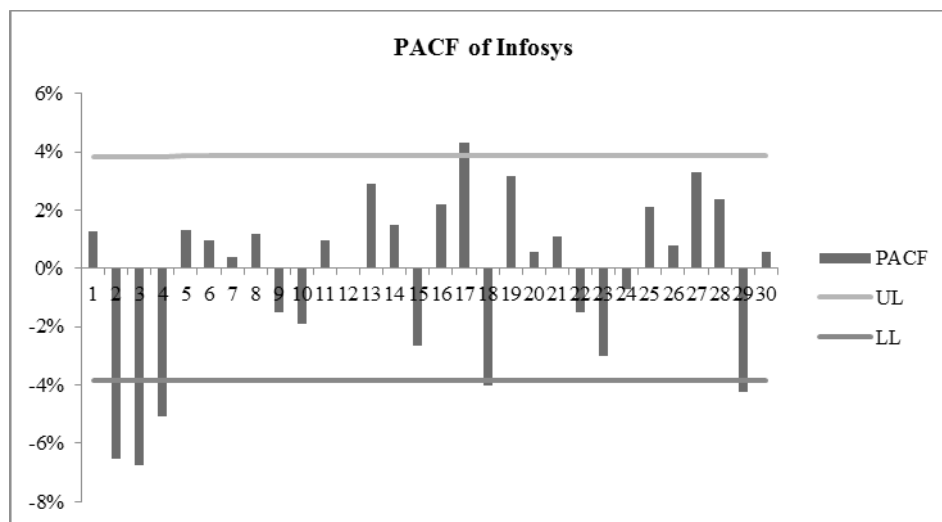


Fig. 3: Partial Autocorrelation for a) TCS, b) Tech Mahindra & c) Infosys

Table 2: ARMA Model Output & Analysis

Model Testing	TCS	Infosys	Tech Mahindra
Fitness Of Good	100%	100%	100%
Noise	False	False	False
Skewness	Negatively Skewed	Negatively Skewed	Positively Skewed
Kurtosis	Leptokurtic	Leptokurtic	Leptokurtic

Table 3: E-GARCH Model Output & Analysis

Model Testing	TCS	Infosys	Tech Mahindra
Long Run Conditional Volatility	2.2%	2.6%	3.01%
Fitness Of Good	100%	100%	100%
Noise	False	False	False
Skewness	Negatively Skewed	Negatively Skewed	Positively Skewed
Kurtosis	Leptokurtic	Leptokurtic	Leptokurtic
Beta	0.69	0.49	0.69

Result

TCS has the least volatility of 2.2 over Infosys and Tech Mahindra whereas Tech Mahindra has the highest volatility. One of the most fascinating aspects of the information technology sector is the very complex market structure that has evolved during the past decade. Companies that want to measure a return on IT investment should develop their own method, as every company is unique. Ignoring this may result in imminent risk of getting incorrect feedback of the evaluation. A true ROI calculation should

be based on real life business scenarios, and there is no single way to calculate a project's potential ROI. It is expected that Indian IT sector will grow at a CAGR of 13.2-15.2% over the next five years. Consequently, the industry size may expand to around \$219-239 billion by FY2019 from \$118 billion projected by NASSCOM for FY2014. Banking sector is the one which is contributing or investing the most in IT Sector. IT Sector is a sector where huge risk is involved, but with high risk, high returns may also come. The method applied using ARMA and GARCH will give the best suited result as discussed

in earlier section as well. Summary sheet for ARMA & GARCH is highlighted in Tables 2 and 3.

Beta is the measure of volatility, beta of 1 means prices of stock will move with the market. A beta of less than one indicates prices will be less volatile than the market and greater than one indicates prices will be more volatile. Thus all the three stocks have beta less than one means less volatile. Fastest growing sectors within various domain-knowledge services, legal services, cloud based services, IT consulting, outsourcing, etc. demand for IT services from US and Europe for India to emerge as the global hub and destination for IT and BPM services by 2020. The sector accounts for 38% of India's services exports.

Conclusion

The study helps to forecast the stock prices of three companies of information technology sector which guides the investors to take informed decisions on the basis of past prices. Most of the business today need information technology not only to work more efficiently but more importantly, to remain competitive in a changing environment. The analysis of the stock market cycles shows that in general over the reference period the bull phases are longer, the amplitude of bull phases is higher and the volatility in bull phases is also higher. The gains during expansions are larger than the losses during the bear phases of the stock market cycles. The bull phase in comparison with its pre-liberalisation character is more stable in the post-liberalisation phase. The results of our analysis also show that the stock market cycles have dampened in the recent past. Volatility has declined in the post-liberalisation phase for both the bull and bear phase of the stock market cycle.

ARMA and E-GARCH are suitable models for the study of the prices and volatility of stocks. The goodness of fit for all the analysis is 1 and this shows that the models are correct in their output and forecasting. Looking at the results it is predicted that the industry is going great, specially the three companies mentioned in the research paper. Apart from the Indian IT sector, IT sector internationally also hold a lot of potential in it from the investment point of view.

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Identification of Customer Clusters using RFM Model: A Case of Diverse Purchaser Classification

Riktesh Srivastava*

Abstract

Competitive world today stresses of having virtuous marketing strategies to appeal new customers while holding longstanding customers. Organisations use instruments to embrace both types of customers, thereby, probing better return on investments and ensuing increasing revenues. The notion of “customer clustering” is used by organisations to categorise diverse fragments of customers and offer them with varied services. The present study takes the four fragments of customers, viz., active, warm, cold, and inactive and does added exploration of these fragments. It was found that these fragments are not enough for defining marketing strategies and need further analysis. The paper magnifies the fragment using RFM analysis then performing clustering on the values obtained from this analysis. This analysis spawns the pertinent rules for each customer segment obtained after clustering.

JEL Codes: G31, G32

Keywords: RFM, Customer Value Pyramid (CVP), Customer Clusters, Clustering without Classification, Clustering with Classification

Introduction

RFM model is an apparatus of clustering customers into 3-dimensions, specifically, recency (R), frequency (F), and monetary value (M). In added arguments, RFM model helps to determine the top 20% of customers, who bring in 80% of revenue. In RFM model, recency (R) is defined as the intermission from the time when the latest consumption happens to the present, frequency (F) is the number of consumption within a certain period, and monetary (M) is the amount of money spent within a certain period. An earlier study showed that customers

with bigger R, F, and M values are more likely to make a new transaction (Wu& Lin, 2002).

In order to group customers and perform analysis, a customer segmentation model-Customer pyramid model is used (Curry & Curry, 2000). Allowance of customer pyramid to model group customers by the revenue they generate is shown in Fig.1 (<http://mnama.blogspot.ae>).

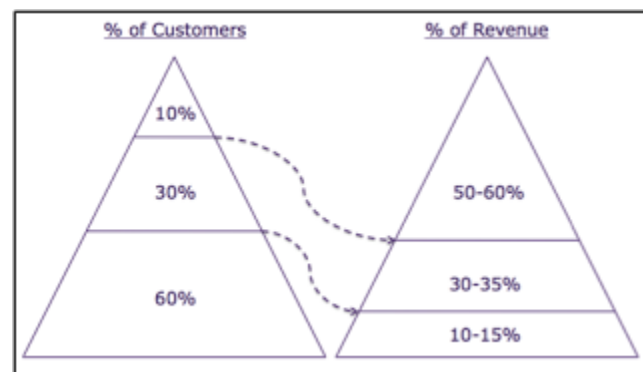


Fig. 1: % of Customers v/s % of Revenue

As stated in Fig. 1, the uppermost 10% of customers epitomizes amid 50-60% of revenue, next 30% embodies 30-35% of revenue. The bottom 60% of customers has awfully low value, and gives less than 15% of total revenue. These three stages of the customer value pyramid can be divided as active, warm, and cold. Added elaboration of the pyramid into 4 dimensions comprises the following four customer types– active, warm, cold, and inactive (<https://lawsonhembree.wordpress.com>).

Both the studies (<http://mnama.blogspot.ae>, <https://lawsonhembree.wordpress.com>) suggest that the customer exhibiting high RFM score should normally conduct more transactions and result in higher revenue. RFM analysis (Im, & Park, 1999; Madeira, 2002) is

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used to further enhance the customer value pyramid into different subsections. As mentioned by Cheng & Chen (2009), there are two opinions on the importance of R, F and M values, while the three parameters are considered equally important in Miglautsch (2000). They are unequally weighted due to the characteristics of industry in Tsai and Chiu (2004). 96 data objects with 1659 observations collected for data analysis in the study adopted the weighted characteristics of R, F and M and further classified the customer clusters into 8 segments.

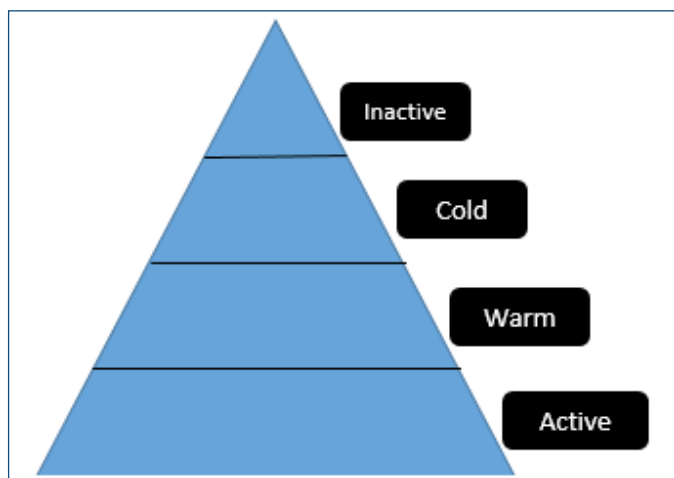


Fig. 2: Four Dimensions of Customer Value Pyramid

The complete paper is organized in 5 sections. Second section exemplifies the data analysis “without further classifications”, labeled as “Clustering without classification”, for R, F and M for all 96 data objects. Third section does the investigation “with further classifications”, named as “Clustering with classification”. Fourth section conducts the revenue analysis of 2015, 2014 and 2013 and gauges the customer clustering of 8 segments and revenue generated. Fifth section accomplishes the paper with recommendations and interpretations.

Clustering without Classification

In clustering without classification, the customer value pyramid is divided in 4 layers, namely, Active, Warm, Cold and Inactive respectively. The output obtained in portrayed in Fig. 3.

There are two important observations from CVP:

1. No customer is inactive.

2. Majority of customers, 84.4% fall under “Active Cluster”, which appears to be a worthy signal for the organisations.

The R, F and M accompanied for the 1659 observations are quantified in Figs.4a, 4b, and 4c, respectively.

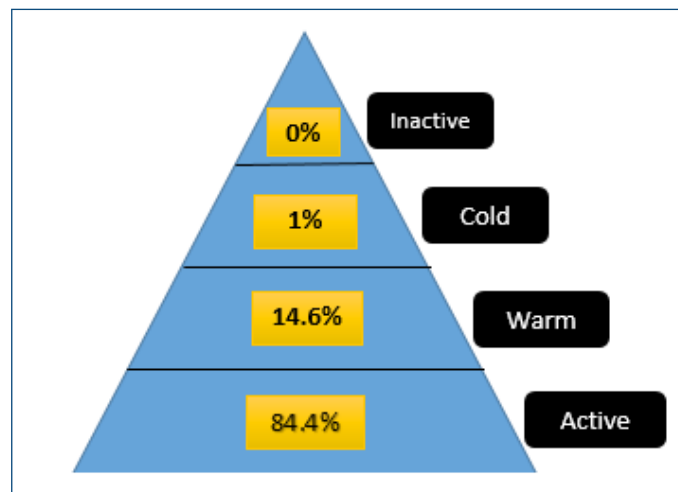


Fig. 3: Clustering Without Classification

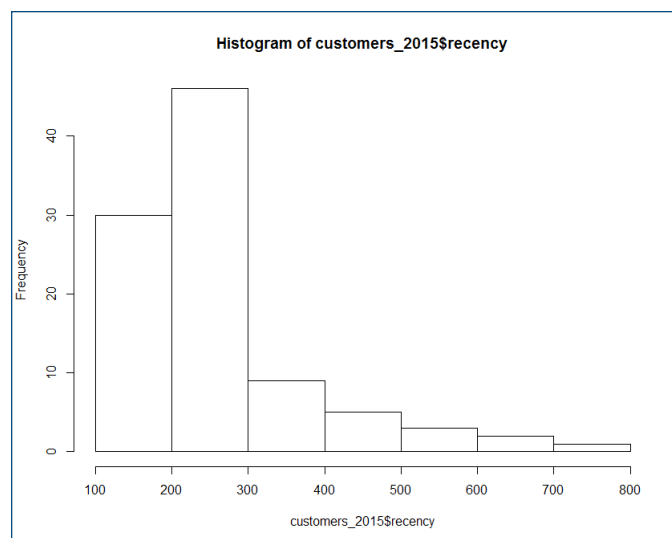


Fig. 4a: Recency for Clustering without Classification

The key observations from Figs. 4a, 4b and 4c are as under:

- Recency between two shopping space is between 100-300 days.
- The frequency of customers is quite high, and falls between 0-25 times.

- Maximum amount spend by customers falls in range of \$900-\$1000.

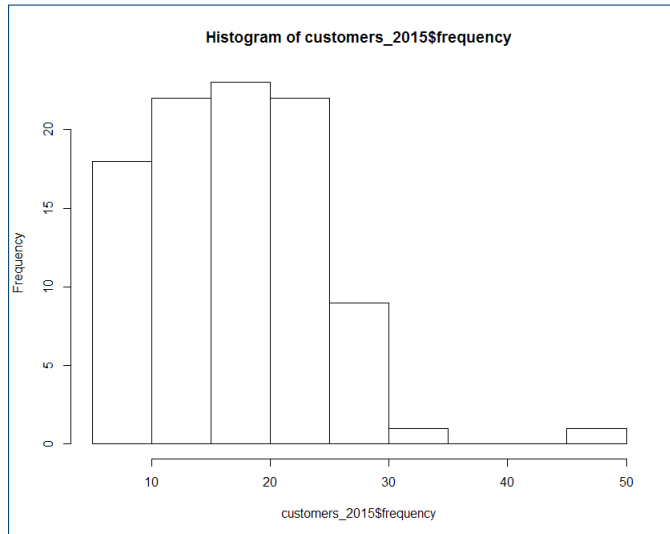


Fig. 4b: Frequency for Clustering without Classification

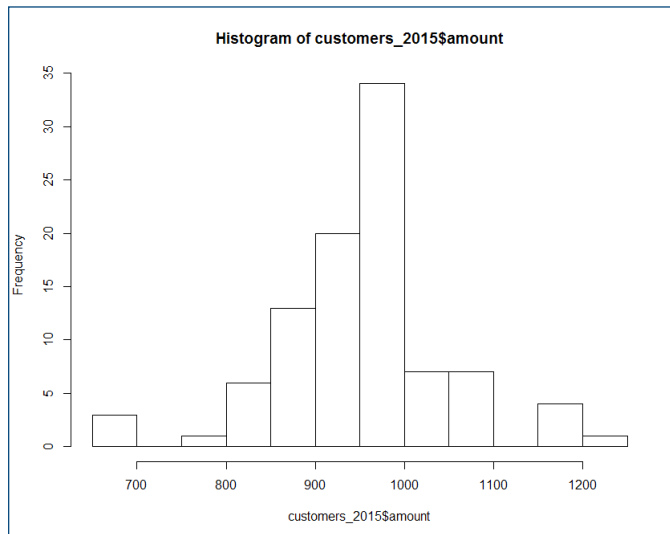


Fig. 4c: Monetary Value for Clustering without Classification

RFM analysis conducted for 4-dimension customer is shown in Fig. 5.

Group.1	recency	first_purchase	frequency	amount
1	active	221.8873	975.8380	17.41975
2	cold	756.8750	908.8750	21.00000
3	warm	466.3036	954.2321	16.21429

Fig. 5: RFM Analysis for Clustering without Classification

The observations are:

- The shopping space between active customers is 221.8873, however, the average amount spend by them is least at \$951.5221.
- The shopping space between cold customers is 756.8750, however, the average amount spend by them is maximum at \$951.5221.

Clustering with Classification

Clustering with classification stretches the improved representation of the different types of customers in “active” and “warm” section, being two most vital categories of customers. The active type of customers is divided into 3 subsections – Active High, Active Low, and New Active, where New Active is the customer whose first purchase is within 365 days. Active High and Active Low are the classifications for the Monetary value (M) more than or less than 100 respectively. The warm type of customers is also alienated into 3 subsections – Active Warm, Active Warm and New Warm, where New Warm is the customer whose first purchase is within 365 days. Active Warm and Active Warm are the classifications for the Monetary value (M) more than or less than 100 correspondingly.

These added classifications are stated on CVP in Fig.6.

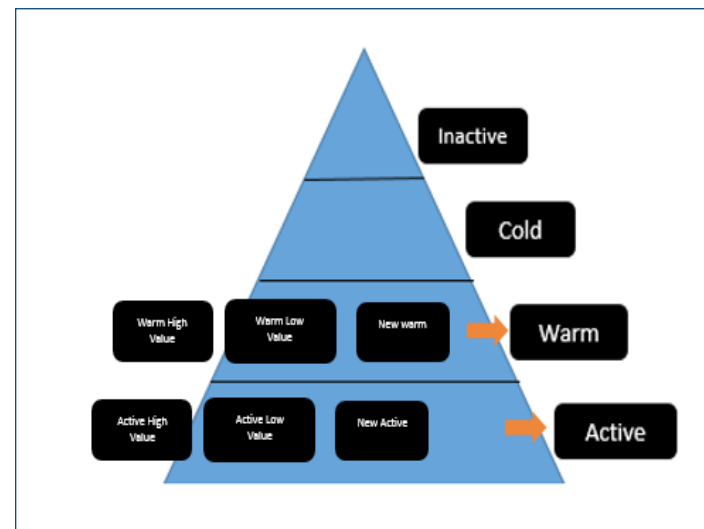


Fig.6: Cluster with Classification

Based on these classifications, trials were again piloted and effects witnessed are declared in Figs.7a, 7b and 7c, respectively.


```
> table(customers_2013$segment)

      inactive      cold  warm high value  warm low value  new warm active high value  active low value
      0          0          0              0              91              0              0
new active
      0
> aggregate(x = customers_2013[, 2:5], by = list(customers_2013$segment), mean)
  Group.1 recency first_purchase frequency amount
1 new warm 375.4245      632.4684  8.648352 951.4591
>
```

Fig. 7a: Outcome of Year 2013 for Cluster with Classification

```
> table(customers_2014$segment)

      inactive      cold  warm high value  warm low value  new warm active high value  active low value
      0          0          0              0              7              84              0
new active
      5
> pie(table(customers_2014$segment), col = rainbow(24))
> aggregate(x = customers_2014[, 2:5], by = list(customers_2014$segment), mean)
  Group.1 recency first_purchase frequency amount
1 new warm 379.58929      610.0179  8.571429 845.6222
2 active high value 64.30357      634.3393 13.440476 960.1452
3 new active 23.87500      143.2750 16.400000 1025.9247
>
```

Fig. 7b: Outcome of Year 2014 for Cluster with Classification

```
> table(customers_2015$segment)

      inactive      cold  warm high value  warm low value  new warm active high value  active low value
      0          1          13              0              1              81              0
new active
      0
> aggregate(x = customers_2015[, 2:5], by = list(customers_2015$segment), mean)
  Group.1 recency first_purchase frequency amount
1 cold 756.8750      908.8750 21.00000 980.7143
2 warm high value 469.1827      979.0288 15.76923 950.8312
3 new warm 428.8750      631.8750 22.00000 951.1364
4 active high value 221.8873      975.8380 17.41975 951.5221
```

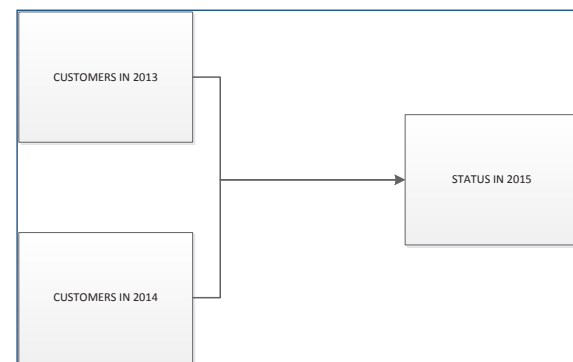
Fig. 7c: Outcome of Year 2015 for Cluster with Classification

These outcomes give the improved perspective of the comprehensive cluster breakdown for three year interval. The foremost results are:

- There is only one category of customer – new warm – in year 2013, indicating no new customer was acquired during this period.
- New customers were acquired in year 2014 and they resulted in maximum monetary value.
- Many customers were “cold” in the year 2015, indicating they are not repeated customers.

Revenue Analysis: Segmenting Database Retrospectively

The segment does retrospective analysis of the customers in 2013 and 2014, and their status in 2015, as shown in Fig. 8 and estimates the revenue created by these sets of customers in 2015. The analysis aids the organisation to outline certain set of policies for customer clusters.

**Fig. 8: Retrospective Analysis of customer status in 2015**

The revenue generated by these sets of customers in 2015 is shown in Fig. 9.

Graphical representations of the revenue generated are also depicted in Figs.10a, 10b and 10c.

<p>Group.1 x</p> <p>4 active high value 4558.951</p> <p>1 cold NA</p> <p>2 warm high value NA</p> <p>3 new warm NA</p>	<p>Group.1 x</p> <p>3 new active 8556.000</p> <p>2 active high value 3662.381</p> <p>1 new warm 2693.571</p>	<p>Group.1 x</p> <p>1 new warm 3587.857</p>
Revenue generated by Customers acquired in 2015 and revenue in 2015	Revenue generated by Customers acquired in 2014 and revenue in 2015	Revenue generated by Customers acquired in 2013 and revenue in 2015

Fig. 9: Revenue Status in 2015

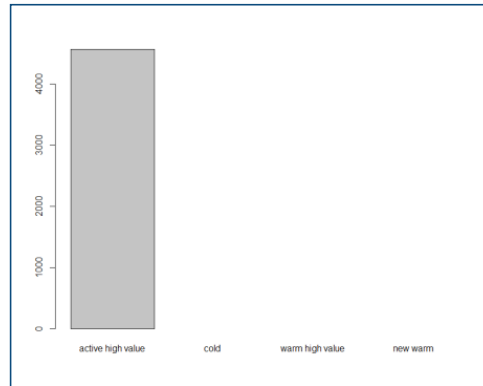


Fig.10a: Revenue Generated by Customers Acquired in 2015 and Revenue in 2015

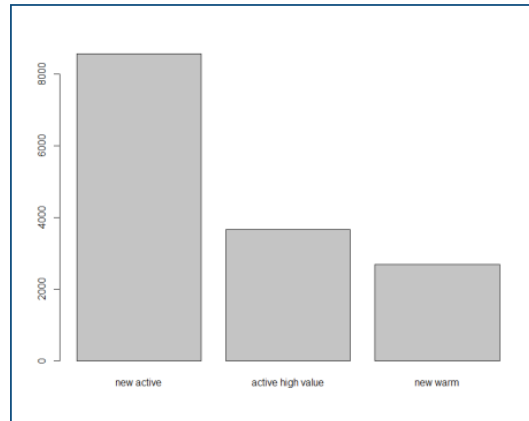


Fig.10b: Revenue Generated by Customers Acquired in 2014 and Revenue in 2015

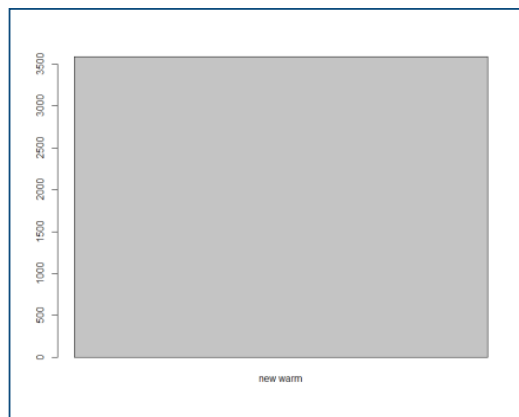


Fig.10c: Revenue Generated by Customers Acquired in 2013 and Revenue in 2015

Recommendations and Conclusion

The study divulges an acumen of the customer analysis and portrays a vital results for the organisations. R programming language is used to scrutinise the data objects and stretches an improved appreciative of the 1659 observations for 96 data objects (customers). These observations are reasonably keys to mount the strategies for customer acquisition. The result exposes that only 1.06% of active customers of 2013 and 2014 get transformed in 2015, which is frightening position for the organisation. Also, only 0.397% of warm category customers of 2013 and 2014 get transformed in 2015. This stipulates that 98.5% target is attained in 2015, and does not assure continual purchase in subsequent years. The company must outline the strategies for customer retaining, which might include announcing loyalty offers, sale deal, or superior deal for these customers.

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Supplier Selection Under Incomplete Preference Information

Mohammad Azadfallah*

Abstract

Supplier selection is a Multiple Attribute Decision Making (MADM) problem which is affected by several conflicting factors as the suppliers' information and performances are usually incomplete and uncertain. Several MADM methods have been proposed for solving this problem, one of which is the Analytic Hierarchy Process (AHP). One of the advantages of this approach is the ability to make incomplete comparisons to get a final priority vector (particularly, in combination with Harker's method). Therefore, calculating priorities with incomplete preference information on alternatives is the aim of this paper. Finally, a numerical example for supplier selection is given to illustrate the application of the proposed method. The main findings of this study confirm the effectiveness of the proposed methods.

Keywords: MADM, AHP, Harker's Method, Incomplete Preference Information, Supplier Selection Problem

Introduction

Companies are profit focused. Traditionally, managers have given strong emphasis to sales revenue - the incoming money. Nowadays, due to fierce competition, increasing price has become a difficult strategic choice. Consequently, growing emphasis has been given to the cost - the outgoing money. Naturally, purchasing as the largest expenses to a company has been receiving an increasingly amount of attention and effort. Since, selecting suitable suppliers is the cornerstone of successful purchasing. However, identifying suitable suppliers is not an easy task. One can argue that it is extremely difficult for any single supplier to excel in all criteria. An actual choice of supplier unavoidable involves trade-off among the attribute levels of different suppliers. Therefore, supplier selection is usually a complex multi-criteria problem involving both quantitative and qualitative elements.

There is no one proven best method in evaluating and selecting suppliers and companies deploy a variety of different approaches. Choosing the best supplier should meet the goal of receiving the right quantity on the right time with the right cost (Jounio, 2013). The AHP is a Multi Criteria Decision Making (MCDM) method that helps the decision maker facing a complex problem with multiple conflicting and subjective criteria (Ishizaka & Labib, 2009). In addition, AHP is a prominent approach in multi criteria decision-making problems; and in practice, it has found widespread application in supplier evaluation and selection problems, alone or in combination with another tool (Jounio, 2013). According to Saaty (2001), the AHP has a systematic procedure for better judgements.

Generally, AHP is based on three basic principles: decomposition, comparative judgements, and hierarchic composition or synthesis of priorities. The decomposition principle is applied to structure a complex problem into a hierarchy of clusters, sub-clusters, sub-sub clusters and so on. The principle of comparative judgements is applied to construct pairwise comparisons of all combinations of elements in a cluster with respect to the parent of the cluster. These pairwise comparisons are used to derive 'local' priorities of the elements in a cluster with respect to their parent. The principle of hierarchic composition or synthesis is applied to multiply the local priorities of elements in a cluster by the 'global' priority of the parent element, producing global priorities throughout the hierarchy and then adding the global priorities for the lowest level elements [the alternatives] (Forman & Selly, 2001). In standard AHP, an eigenvector (EV) method is used for deriving weights from local matrices; the EV is called the prioritisation method, and the computational procedure is consequently called optimisation. After local weights are calculated at all levels of the hierarchy, a synthesis consists of multiplying the criterion-specific weight of the alternative with the corresponding criterion

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weight and summing up the results to obtain composite weights of the alternative. With respect to the goal, this procedure is unique for all alternatives and all criteria. If all comparisons are performed properly by the DM, then AHP synthesis is straightforward. However, if the DM for various reasons fails to make some judgements, then there are empty cells in the corresponding local matrices. The first case can be treated as decision making with complete information, and the other case with incomplete information (Srdjevic, Srdjevic, & Blagojevic, 2014). The latter is preferred in this paper. Therefore, in this study, the AHP method in combination with Harker's method applied to solve the supplier selection problem under incomplete preference information.

The paper is organised as follow. In the second section, the literature and in the third section, the proposed approach is discussed. Numerical example is provided in the next section. The paper is concluded in the fifth and the last section.

Literature Review

In some real problems, it is impossible or difficult to have comparisons of some pairs of alternatives. Let us call such cases incomplete AHP. It is very important to estimate incomplete comparisons data to have alternatives weights (Gao, Zhang, & Cao, 2010). Several solutions for the above problem are possible, too like Harker's method, Van Uden's method, etc. Here, we will mention some of them. Harker (1987) explored various methods for reducing the complexity of the preference eliciting process from the incomplete pairwise comparisons in the AHP technique. The theory of a method based upon the graph-theoretic structure of the pairwise comparison matrix and the gradient of the right Perron vector is developed, and simulations of a series of random matrices are used to illustrate the properties of this approach. Wedley (1993) developed a multiple regression equations methodology for predicting the consistency index and consistency ratio when pairwise comparisons are incomplete. Wedley *et al.* (1993) studied the effect of different reference items for the first (n-1) pairwise comparisons of the incomplete AHP. The empirical results show that significantly greater initial accuracy is achieved if the items are ranked and the lowest ranked item is used as a common referent for the first (n-1) comparisons. Fan and Ma (1999) proposed a new approach to solve the MADM problem with incomplete preference information on alternatives. The approach is

based on an optimisation model which can be used to assess attribute weights and then to select the most desirable alternative. Lu and Yang (2005) focused on the impact of incomplete information that buyers always possessed on their choice behavior in air transportation market. Fedrizzi and Giove (2006) introduced a new method for calculating the missing elements of an incomplete matrix of pairwise comparison values for a decision problem. So, the matrix is completed by minimising a measure of global inconsistency, thus obtaining a matrix, which is optimal from the point of view of consistency with respect to the available judgements. Wedley (2006) enlarged the spanning tree, then put into an expanded matrix with incomplete comparisons and solved using Harker's method. Alonso, Herrera-Viedma, Chiclana, & Herrera (2009) presented two different tools that can be used to solve GDM (Group Decision Making) problems where the experts give their preferences by means of incomplete fuzzy preference relations. Gao *et al.* (2010) proposed a new method for ranking estimation in incomplete AHP. So that, new least square method is translated into linear system and minimax method, and absolute deviation method are translated into linear programming. It is shown that three proposed methods have fast convergence and smaller computational complexity. Chuang, Kao, Cheng, & Chou (2012) investigated the relationship between the incomplete information and the compromise effect in different choice scenarios. The main findings were that consumers are more likely to choose the middle option when they have incomplete information than when they have complete information. Carmo *et al.* (2013) addressed the aggregation of individual priorities (AIP) in incomplete hierarchies, in AHP. The result shows that only arithmetic mean aggregation of individual priorities is suitable to be used when incomplete hierarchy is considered. Srdjevic *et al.* (2014) proposed a method to fill the gaps in the pairwise comparison matrices generated through elicitation of the DM's semantic judgements.

This paper focuses on the application of the AHP method in combination with Harker's method for solving a supplier selection problem, under incomplete information. In the next section, the proposed method will be considered.

Proposed Approach

A brief discussion of AHP and Harker's method is provided in this section.

Analytic Hierarchy Process (AHP)

In the Analytic Hierarchy Process, first, elements are compared in the form of pair and paired comparison matrix, then formed by use of this matrix to calculate the relative weights of elements to tally a paired comparison matrix shown in the following form in which a_{ij} is the preference of element i to element j . now with the determination of a_{ij} , we want to gain weights W_i of elements

$$A=[a_{ij}], \quad i, j=1,2,\dots,n. \quad (1)$$

Each paired comparison matrix may be consistent ($a_{ij}=W_i/W_j$) or inconsistent ($a_{ij} \neq W_i/W_j$). In the state, that the matrix is consistent and calculating weight W_i is simple and gained from normalisation of the elements of each column. But in the state that matrixes are inconsistent, four main approaches will be presented for calculation:

1. Least squares method
2. Logarithmic least squares method
3. Eigenvector methods
4. Approximation methods.

Now we explain one of the above methods that we have used for achieving weights in the presented method. In this method, W_i is a determinant in a way that the following relationship is true.

$$\begin{aligned} a_{11}W_1 + a_{12}W_2 + \dots + a_{1n}W_n &= \lambda \cdot W_1 \\ a_{21}W_1 + a_{22}W_2 + \dots + a_{2n}W_n &= \lambda \cdot W_2 \\ &\vdots \\ a_{n1}W_1 + a_{n2}W_2 + \dots + a_{nn}W_n &= \lambda \cdot W_n \end{aligned} \quad (2)$$

Here a_{ij} is preference of i -th element to j -th, W_i is weight of i -th element, and λ is a constant number. This method is one kind of mean that Harker calls it as possible mean in a different way because in this way the weight of i -th element (W_i) according to the above definition is equal to:

$$W_i = 1/\lambda \cdot a_{n1}W_1 + a_{n2}W_2 + \dots + a_{nn}W_n = \lambda \cdot W_n \quad (3)$$

$i=1,2,\dots,n$

We can write the above simultaneous equations as follows:

$$A \cdot w = \lambda \cdot w \quad (4)$$

in which A is a paired comparison matrix $\{ \text{mean } A=[a_{ij}] \}$, w is a weight vector, and λ is a scalar (number). According to the definition, this relationship is among one matrix (A), vector (W) and (λ) number, it has been said that w is a special vector and λ is a special amount (Lotfi *et al.*, 2012).

Harker's Method

Harker method is based on the following idea. If (i, j) – component is missing, put the artificial value W_i/W_j into the vacant component to construct a complete reciprocal matrix A (W). Then consider the eigen system problem:

$$A(w)w = \lambda w \quad (5)$$

Formally, Harker's method is written as follow. Given incomplete matrix $A=(a_{ij})$, define the corresponding derived reciprocal matrix $\tilde{A}=(\tilde{a}_{ij})$ by:

$$\begin{aligned} 1+m_i & \quad \text{if } i=j \\ \tilde{a}_{ij}=0 & \quad \text{if } a_{ij} \text{ is missing} \\ a_{ij} & \quad \text{other wise} \end{aligned} \quad (6)$$

where m_i denotes the number of missing components in the i -th row (Gao *et al.*, 2010). In other words, enter zero for any missing judgement in that matrix, and add the number of missing judgements in each row to the diagonal element in the row, producing a new matrix \tilde{A} . then, calculate the weight W (Saaty, 2000):

$$\lim_{k \rightarrow \infty} A^k / e^t A^k = cw \quad (7)$$

where e is the column vector unity, e^t is its transpose, and c is a positive constant.

Numerical Example

To demonstrate the application of the proposed approach in supplier selection context, we use the dataset from Benyoucef, Ding, & Xie (2003) [problem with a known composite answer; Tables 1-13].

Table 1: Pairwise Comparisons of Evaluation Criteria

Objective	Pricing structure	Delivery	Quality	Service	Weights
Pricing structure	1	3	1	3	0.40
Delivery	1/3	1	1/3	1	0.13
Quality	1	3	1	1/2	0.26
Service	1/3	1	2	1	0.21

Table 2: Pairwise Comparisons of Criterion Dimensions - 1

Delivery	Timeliness	Cost	Weights
Timeliness	1	5	0.83
Cost	1/5	1	0.17

Table 3: Pairwise Comparisons of Criterion Dimensions - 2

Quality	Quality level	Cost	Weights
Quality level	1	1/5	0.17
Cost	5	1	0.83

Table 4: Pairwise Comparisons of Criterion Dimensions - 3

Service	Personnel	Facilities	R&D	Capability	Weights
Personnel	1	1	1	3	0.32
Facilities	1	1	2	1/2	0.24
R&D	1	1/2	1	1/2	0.17
Capability	1/3	2	2	1	0.26

Table 5: Pairwise Comparisons of Suppliers A, B, and C - 1

Price structure	A	B	C	Weights
A	1	1	1/5	0.16
B	1	1	1/3	0.18
C	5	3	1	0.66

Table 6: Pairwise Comparisons of Suppliers A, B, and C - 2

Timeliness	A	B	C	Weights
A	1	5	3	0.66
B	1/5	1	1	0.16
C	1/3	1	1	0.18

Table 7: Pairwise Comparisons of Suppliers A, B, and C - 3

Delivery cost	A	B	C	Weights
A	1	1/3	2	0.23
B	3	1	5	0.65
C	1/2	1/5	1	0.12

Table 8: Pairwise Comparisons of Suppliers A, B, and C - 4

Quality level	A	B	C	Weights
A	1	1/2	1/3	0.16
B	2	1	1/2	0.30
C	3	2	1	0.54

Table 9: Pairwise Comparisons of Suppliers A, B, and C - 5

Quality cost	A	B	C	Weights
A	1	1	1/2	0.25
B	1	1	1/2	0.25
C	2	2	1	0.50

Table 10: Pairwise Comparisons of Suppliers A, B, and C - 6

Personnel	A	B	C	Weights
A	1	1/5	2	0.20
B	5	1	3	0.65
C	1/2	1/3	1	0.15

Table 11: Pairwise Comparisons of Suppliers A, B, and C - 7

Facilities	A	B	C	Weights
A	1	1	1	0.33
B	1	1	1	0.33
C	1	1	1	0.33

Table 12: Pairwise Comparisons of Suppliers A, B, and C - 8

R&D	A	B	C	Weights
A	1	1/3	1/3	0.14
B	3	1	1	0.43
C	3	1	1	0.43

Table 13: Pairwise Comparisons of Suppliers A, B, and C - 9

Capability	A	B	C	Weights
A	1	3	1	0.42
B	1/3	1	¼	0.12
C	1	4	1	0.46

The result is as follow:

$$\begin{bmatrix} 0.157 \\ 0.185 \\ 0.685 \end{bmatrix} \cdot 0.40 + \begin{bmatrix} 0.586 \\ 0.240 \\ 0.174 \end{bmatrix} \cdot 0.13 + \begin{bmatrix} 0.230 \\ 0.259 \\ 0.507 \end{bmatrix} \cdot 0.26 + \begin{bmatrix} 0.280 \\ 0.400 \\ 0.320 \end{bmatrix} \cdot 0.21 = \begin{bmatrix} 0.260 \\ 0.250 \\ 0.490 \end{bmatrix}$$

From the above results, it can be concluded that, the ranking is as follow:

$$C > A > B$$

In this section assume that, DM for each reasons fails to make some judgements (for instance, for Table 1, 4, 5, 7, 8, 10, 12, and 13, respectively). Therefore, there are empty cells in the corresponding local matrices (Table 14-21) i.e. for Table 1 (based on formula No. 6 and 7), we have (Table 14):

Table 14: Incomplete Pairwise Comparisons of Evaluation Criteria (Based on Table 1)

Objective	Pricing structure	Delivery	Quality	Service	Weights
Pricing structure	1	?	1	3	0.386
Delivery	?	1	?	1	.174
Quality	1	?	1	½	.217
Service	1/3	1	2	1	.223

$$A = \begin{bmatrix} 2 & 0 & 1 & 3 \\ 0 & 3 & 0 & 1 \\ 1 & 0 & 2 & 1/2 \\ 1/3 & 1 & 2 & 1 \end{bmatrix} = A^1; e = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$A^1 \cdot e = \begin{bmatrix} 6 \\ 4 \\ 3.50 \\ 4.33 \end{bmatrix}; e^T \cdot A^1 \cdot e = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 6 \\ 4 \\ 3.50 \\ 4.33 \end{bmatrix} = 17.833$$

Then in first iteration:

$$W^1 = A^1 \cdot e / e^T \cdot A^1 \cdot e = (6/17.833=0.336, 4/17.833=0.224, 3.50/17.833=0.196, \text{ and } 4.33/17.833=0.243).$$

Similarly (details are omitted):

$$W^2 = (0.369 \quad 0.211 \quad 0.196 \quad 0.224)$$

$$W^3 = (0.375 \quad 0.200 \quad 0.204 \quad 0.222)$$

$$W^4 = (0.377 \quad 0.192 \quad 0.208 \quad 0.223)$$

$$W^5 = (0.380 \quad 0.186 \quad 0.211 \quad 0.223)$$

$$W^6 = (0.382 \quad 0.182 \quad 0.213 \quad 0.223)$$

$$W^7 = (0.384 \quad 0.179 \quad 0.214 \quad 0.223)$$

$$W^8 = (0.385 \quad 0.177 \quad 0.215 \quad 0.223)$$

$$W^9 = (0.385 \quad 0.176 \quad 0.216 \quad 0.223)$$

$$W^{10} = (0.386 \quad 0.175 \quad 0.216 \quad 0.223)$$

$$W^{11} = (0.386 \quad 0.174 \quad 0.217 \quad 0.223)$$

Finally after 12 iterations:

$$W^{12} = (0.386 \quad 0.174 \quad 0.217 \quad 0.223)$$

As can be seen, the process has convergence in twelfth iteration and the calculation was stabled. In other words, W^{12} is the final solution (the last column of Table 14).

Similarly;

Table 15: Incomplete Pairwise Comparisons of Evaluation Criteria (based on table 4)

Service	Pricing structure	Delivery	Quality	Service	Weights
Pricing structure	1	1	1	3	0.334
Delivery	1	1	2	½	.240
Quality	1	1/2	2	0	.185
Service	1/3	2	0	2	.241

Table 16: Pairwise Comparisons of Suppliers – 1 (based on Table 5)

Price structure	A	B	C	Weights
A	2	0	1/5	0.131
B	0	2	1/3	0.217
C	5	3	1	0.652

Table 17: Pairwise Comparisons of Suppliers – 2
(based on Table 7)

Delivery cost	A	B	C	Weights
A	2	1/3	0	0.217
B	3	1	5	0.652
C	0	1/5	2	0.131

Table 18: Pairwise Comparisons of Suppliers - 3
(based on Table 8)

Quality level	A	B	C	Weights
A	1	1/2	1/3	0.167
B	2	2	0	0.334
C	3	0	2	0.449

Table 19: Pairwise Comparisons of Suppliers - 4
(based on Table 10)

Personnel	A	B	C	Weights
A	2	0	2	0.334
B	0	2	3	0.500
C	1/2	1/3	1	0.167

Table 20: Pairwise Comparisons of Suppliers - 5
(based on Table 12)

R&D	A	B	C	Weights
A	3	0	0	0.333
B	0	2	1	0.333
C	0	1	2	0.333

Table 21: Pairwise Comparisons of Suppliers - 6
(based on Table 13)

Capability	A	B	C	Weights
A	2	3	0	0.376
B	1/3	1	1/4	0.125
C	0	4	2	0.499

The new result is as follow:

$$\begin{bmatrix} 0.131 \\ 0.217 \\ 0.652 \end{bmatrix} 0.386 + \begin{bmatrix} 0.584 \\ 0.240 \\ 0.176 \end{bmatrix} 0.174 + \begin{bmatrix} 0.236 \\ 0.264 \\ 0.500 \end{bmatrix} 0.217 + \begin{bmatrix} 0.344 \\ 0.339 \\ 0.317 \end{bmatrix} 0.223 = \begin{bmatrix} 0.280 \\ 0.258 \\ 0.461 \end{bmatrix}$$

From the above results, it can be concluded that, the ranking is as follow:

$$C > A > B$$

A comparison of the test results is given in Table 22.

Table 22: Comparison of Results

Method	Priorities
AHP (under complete preference information)	C > A > B .490 .260 .250
AHP and Harker's method (under incomplete preference information)	C > A > B .461 .280 .258

Comparative results shown in Table 22 indicate that results obtained by AHP (under complete preference information) were not different from those obtained using the AHP and Harker's method (under incomplete preference information). Moreover, these results implicitly indicate the effectiveness of the proposed models.

Concluding Remarks

In this paper, we proposed a model for supplier evaluation using AHP and Harker's method to evaluating and ranking supplier under incomplete preference information (the results were tested by the example with a known composite answers). Then a comparative analysis is performed (Table 22). The results indicate that the ranks obtained by the AHP and AHP-Harker's method are not different. Therefore, the findings in this paper confirm the effectiveness of proposed method (because these results implicitly indicate the accuracy of the applied methods). In addition, further research can apply this proposed approach to other managerial issues or compared with another method to estimate incomplete comparison data.

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Hurdle Rate Analysis for Indian Power Projects

Abhishek Jha*, Gaurav Aggarwal**

Abstract

Hurdle rate is the minimum rate of return required by the investor. It has generally been observed that setting up of a hurdle rate in power generation projects is done arbitrarily purely on the basis of rules of thumb in Indian scenario. The present study is an effort to empirically link various variables utilised for making judgements on values of hurdle rate and the hurdle rate itself. The various variables used in the study are project's Weighted Average Cost of Capital (WACC), project default premium, project's unconventional risk factor, project's strategic discount factor, project's tenure and expected inflation rate. Various tests were conducted to check suitability of application of ordinary multiple regression. Finally, robust regression was found suitable to be applied depending on the test results. In addition, stepwise ordinary multiple regression was applied to find the relative significance of each applied independent variable. Suggestions are also provided on establishing a robust system of identification of unconventional risk factors as well as strategic discount factor. Lastly, a model has been developed and the relative significance and ways of application of various independent variables have been discussed.

JEL Codes: G31, G32

Keywords: Hurdle, Budgeting, Robust, Regression, Heteroskedasticity, WACC

Introduction

Concerns have been raised about financial performance of public sector undertakings or PSUs in India. It is believed that sound capital budgeting practices can have a sound effect on improving the performance of PSUs in India. It is also widely assumed that private sector enterprises usually perform better when it comes to making capital budgeting decisions. The present study ascertains the hurdle rate estimation of power projects and then identifies loopholes in the procedures. Conventionally, power

generation projects have used certain thumb rules to estimate hurdle rates. These thumb rules are often easy to measure but lead to various inaccuracies. Overinvestment or underinvestment problems are often reported from power projects. Reports from other industries have suggested that they have started using more theoretically correct models. These models have been studied to provide comparatively more accurate measurements of hurdle rates given the assumption that input data is of high quality. Improvement in methods of data capture has led to better quality and quantity of input data for these models. Trends have been identified to move away from traditional models to more accurate models.

Considerable empirical findings have suggested that there is something unusual about capital budgeting methods and the very process of setting up of discount rates for projects. Firstly, a large number of firms still use unsophisticated evaluation methods of capital budgeting. Different reasons for the non-application of sophisticated capital budgeting methods and general overuse of 'rules of thumb' in investment evaluation have been suggested by many authors. These include limiting the tendency of excess optimism among CEOs and agency problems which is associated with project approval and limited availability of managerial and organisational capital. In addition, few other reasons have been listed as non-explicit inclusion of elements derived from valuation of real option, political risk considerations and rationality bounded by practical limitations. Other papers have even related the education of the CFO to the use of sophisticated capital budgeting methods.

Literature Review

Driver and Temple (2002) studied the relationship between hurdle rate and discount rate so that inference can be made on investment behaviour of different firm characteristics. Existence of strategic options led to the

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lowering of hurdle rate as compared to discount rate. Risk enhances hurdle rate to some extent. Brigham (1975) emphasized the significance of frequent hurdle rate revisions. In addition, it was identified that different projects of the same firm has different riskiness measures and hence, a firm should use more than one hurdle rate for different projects and scenarios. Antle and Eppen (1985) found out that companies generally keep hurdle rates strictly in excess of WACC. Through this policy certain level of capital rationing is also performed. Meier and Tarhan (2007) showed that the self-proclaimed WACC that financial managers in reputed firms use to discount cash flows exceeds computed WACC by 5.3% to 7.5%, which depends on the equity premium assumption. It was also found that the hurdle rate premium is positively correlated with high growth opportunities and the financial soundness and health of firms, and negatively correlated with the goodness of fit of the beta estimation models. Finally, they justified the excess value of hurdle rate over and above the WACC utilised by them. Brunzell, Liljeblom and Vaihekoski (2013) studied application of capital budgeting methods and hurdle rate determination among Nordic companies. They observed empirically that the hurdle rate used by the firms tend to be higher than those suggested by economic theory. If companies used less sophisticated methods of capital budgeting, they tend to use higher hurdle rate premiums.

Calandro, Gates, Madampath and Ramette (2015) highlighted how strategic factors impact upon the hurdle rates and that how finance executives are ruling the fact that no readily accepted models are available to estimate hurdle rates. Chen and Jiang (2004) showed that hurdle rates should be higher if project scales are larger and the project is riskier. In addition, more information collection effort from manager requires setting a higher hurdle rate. Block (2005) showed that there exists a somewhat unanimity among industries in using WACC as a hurdle rate. Public utility industries are the major exception where preference is given to using return on stockholder's equity as the primary metric. Lofgren, Millock and Nauges (2007) highlighted the lack of hurdle rate estimates for pollution abatement investments although sufficient data is available. Hence, they have called for development of models based on econometric approaches applied on observed data. In addition, uncertainty has been observed to increase hurdle rates. Kruger, Landier and Thesmar (2015) identified the fallacy in using the same WACC discount rate for all projects by a firm. Also, the costs

involved in using multiple discount rates may not purely cognitive or computational. Use of multiple discount rates might increase the scope of politicking and gaming of the capital budgeting process. Jagannathan, Meier and Tarhan (2011) found that managers involved in capital budgeting systematically add a hurdle premium to their CAPM based cost of capital. The size of this premium was also found to be substantial. A model of hurdle rates was developed which used nonlinear WACC and other linear variables that proxy for the option to wait. It was determined through a zero intercept that managers do not use higher hurdle rates to compensate for optimistic cash flow projections. While, both cost and capital and hurdle premium components were found to be significant, cost of capital can only explain only 10% of the variation in hurdle rates whereas proxies for the option to wait explain 35%. It was also found that since hurdle premium varies substantially more than the cost of capital across firms, it actually hides the relationship between hurdle rate and CAPM beta.

Research Methodology

In the present study, an attempt has been made to develop a model by which we can constructively determine hurdle rate in a scientific way which is free from biases encountered in practical life. Effort has been made to collect data from higher level managers involved in making investment decisions of power generations projects. Firstly, efforts have been made to understand the practical approaches followed by power generation firms in estimating hurdle rates. Then, information is sought about the practical suitability and ease of applying these approaches. Thereafter, an attempt has been made to develop a model which provides a fairly accurate value of hurdle rate while not compromising on information of project default, project strategic importance, expected inflation and risk, size and tenure of the project. Primary data are collected from power generation firms on all these parameters. Hurdle rate is chosen as our dependent variable whose values are collected from experts from these power generation firms as well as other experts from banks and consulting industry so as to remove general biases. Getting accurate values of hurdle rates is the most basic essence of our model. Finally, regression is applied and a model is generated which is as shown below:

Hurdle Rate = a*Project WACC + b*Project Default Premium + c*Project Unconventional Risk Factor+

$$d * \text{Project Strategic Importance Factor} + e * \text{Project Tenure} + f * \text{Expected Inflation Rate} \quad (1)$$

Hurdle rate is defined as “the minimum rate of return on a project or investment required by the investor”. It may or may not be equal to the project’s WACC as the former is the return expected by shareholders and long term creditors. Firms typically want a higher return to compensate for other factors such as unconventional risk.

Project’s WACC (Weighted Average Cost of Capital) is defined as “the rate of return that a company is expected to pay on average to all its security holders and long term creditors to finance its assets.

Project default premium is additional return expected by a power project as a safety factor.

Project unconventional risk is the ordinal value of unconventional risk inherent in the project expressed on a scale of 0 to 10.

Project strategic importance discount is the ordinal value of strategic importance acknowledged by the power project expressed on a scale of 0 to 10.

Project tenure is the expected duration of successful and profitable operation of the power project. “Useful life” is another term which may be used for the variable. It is expressed in number of decades.

Expected inflation rate is the expected average rate of inflation over the project tenure or its “useful life”. It is expressed as a fraction and not as a percentage.

Effort has been made to identify different independent variables of ratio or ordinal nature which can be used to arrive at a hurdle rate.

Hypothesis

Various hypotheses have been analysed for the study which are as mentioned under:-

- Project’s WACC has no correlation with hurdle rate.
- Project default premium has no correlation with hurdle rate.
- Project unconventional risk has no correlation with hurdle rate.
- Project strategic importance discount has no correlation with hurdle rate.

- Project tenure has no correlation with hurdle rate.
- Expected inflation rate has no correlation with hurdle rate.

Data Analysis

Appropriate values of all discussed variables were collected beforehand. Care is taken such that the value of each variable is as close as possible to the actual value. As we can see the nature of our variables, records of actual values of these variables is not even diligently maintained by the industry experts. Hence, first information is sought by industry experts and the same is confirmed by banks and consulting firms. Data have been collected and confirmed for 123 power generation projects. Data for other power projects which were unreliable and unconfirmed were not included in our study. Linear multiple regression was first sought to be applied for the study.

Before applying multiple linear regression, several assumptions have to be checked in which are as mentioned below:-

- Independence of errors (residuals)
- A linear relationship between predictors and dependent variable.
- Homoscedasticity of residuals (Equal Error Variance)
- No Multicollinearity.
- No significant outliers or influential points.
- Errors (Residuals) are normally distributed.

Model Summary ^a					
Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Durbin-Watson
1	.994 ^a	.988	.987	.00235	1.968

a. Predictors: (Constant), Inflation, DefaultPremium, RiskFactor, WACC, Tenure, ImportanceDiscount
b. Dependent Variable: HurdleRate

Fig. 1: Durbin Watson Statistic of Multiple Linear Regression

As we can see in Fig.1, Durban-Watson statistic is 1.968 which is very close to 2. Hence, we can safely assume that there is independence of residuals and they are not auto correlated.

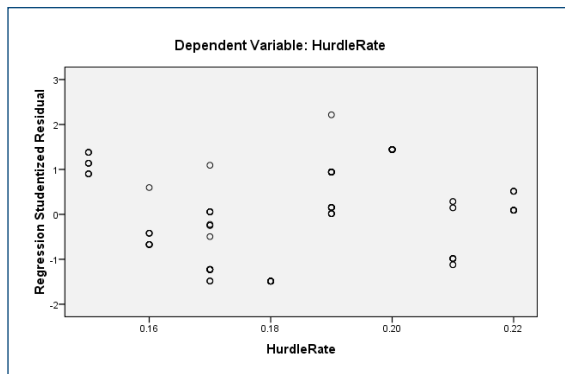


Fig. 2: Plot of Studentised Residual versus Unstandardised Hurdle Rate

As we can see in Fig.2, we have a horizontal band. Hence, it can be safely assumed that the relationship between the dependent and independent variables is likely to be linear. The partial regression plots are also drawn between each independent variable and the dependent variable. These are as presented in Fig. 3.

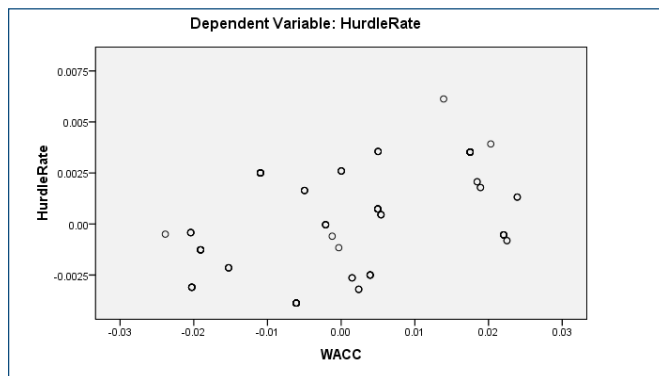


Fig. 3: Plot of Hurdle Rate versus WACC

As we can see from Fig.3, there appears to be a somewhat linear relationship between hurdle rate and WACC. The approximation line doesn't appear to tend upwards or downwards.

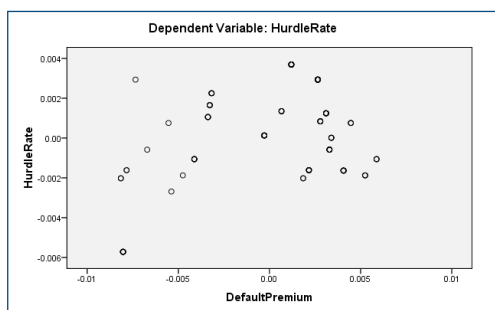


Fig. 4: Plot of Hurdle Rate versus Default Premium

As we can see from Fig.4, there appears to be a somewhat linear relationship between hurdle rate and default premium. The approximation line ignoring one outlier in the bottom doesn't appear to tend steely upwards or downwards.

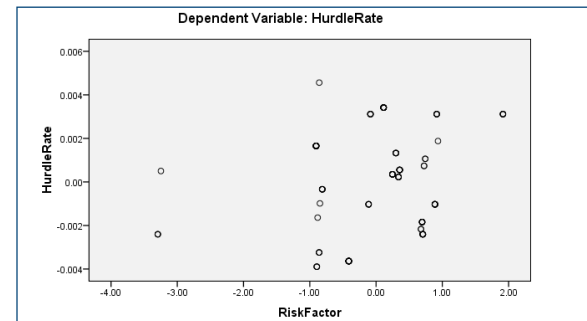


Fig. 5: Plot of Hurdle Rate versus Risk Factor

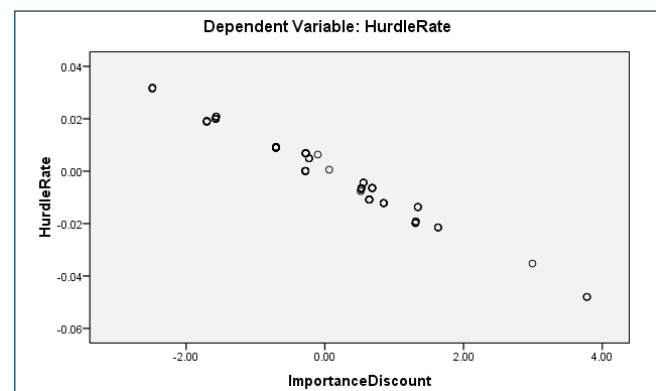


Fig. 6: Plot of Hurdle Rate versus Importance Discount

As we can see from Fig.6, there appears to be a clear linear relationship between hurdle rate and importance discount. Importance discount appears to be negatively correlated with hurdle rate. As importance discount increases, hurdle rate decreases linearly.

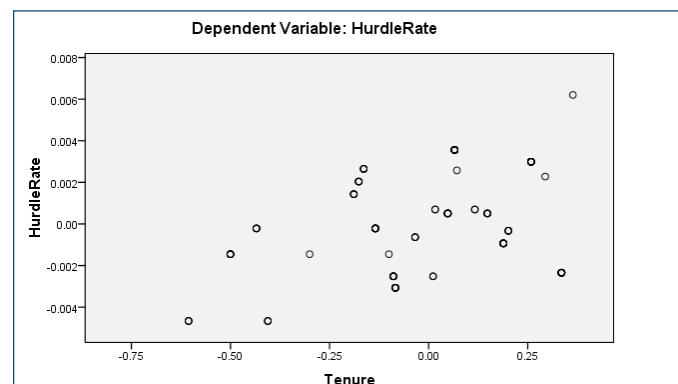


Fig. 7: Plot of Hurdle Rate versus Tenure

As we can see from Fig.7, there appears to be a somewhat linear relationship between hurdle rate and tenure. There appears to be a positive relationship between the two variables. The approximation line tends to uniformly rise slightly upwards and doesn't tend to bend upwards or downwards.

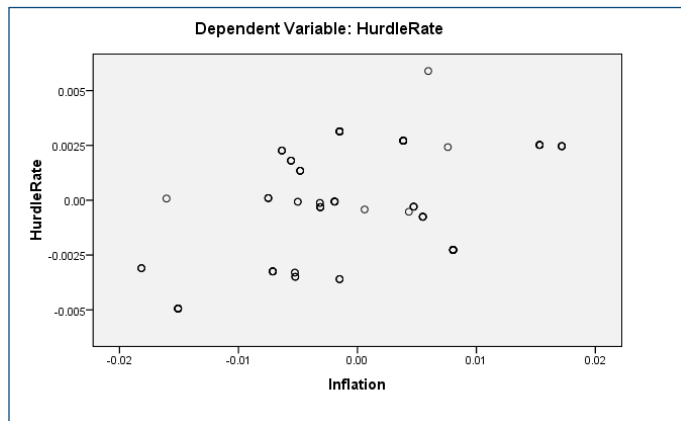


Fig. 8: Plot of Hurdle Rate versus Inflation

As we can see from Fig.8, there appears to be a somewhat linear relationship between hurdle rate and inflation. There appears to be a positive relationship between the two variables. The approximation line tends to uniformly rise slightly upwards and doesn't tend to steeply bend upwards or downwards.

Hence, we can safely proceed towards the analysis of tests of heteroskedasticity. Applying Breusch Pagan Test and Koenker Test on the available data yielded the following results as shown in Fig.9.

Sample size (N)	123
Number of predictors (P)	6
Breusch-Pagan test for Heteroscedasticity (CHI-SQUARE df=P)	9.751
Significance level of Chi-square df=P (H0:homoscedasticity)	.1355
Koenker test for Heteroscedasticity (CHI-SQUARE df=P)	23.698
Significance level of Chi-square df=P (H0:homoscedasticity)	.0006

Fig. 9: Breusch Pagan and Koenker Test Results

As we can see in Fig.9, we fail to reject heteroskedasticity in case of Breusch Pagan Test but reject it in the case of Koenker Test. Hence, for safety reasons we should not proceed ahead with application of multiple linear regression as such. Robust regression might be a better alternative to use. Checking for multicollinearity, we have the information as displayed in Fig.10.

As seen in Fig.10, the VIF values are less than 2. Hence, we can safely assume that there is no multicollinearity. Checking for normality of residuals yielded the results as shown in Fig.11.

Coefficients ^a										
Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	95% Confidence Interval for B		Collinearity Statistics	
		B	Std. Error	Beta			Lower Bound	Upper Bound	Tolerance	VIF
1	(Constant)	.198	.004		49.807	.000	.191	.206		
	WACC	.077	.016	.057	4.981	.000	.047	.108	.782	1.279
	DefaultPremium	.288	.050	.064	5.724	.000	.188	.388	.830	1.205
	RiskFactor	.001	.000	.034	2.448	.016	.000	.001	.541	1.848
	ImportanceDiscount	-.013	.000	-1.012	-72.339	.000	-.013	-.012	.527	1.899
	Tenure	.003	.001	.046	3.642	.000	.001	.005	.644	1.553
	Inflation	.142	.025	.074	5.722	.000	.093	.191	.620	1.613

a. Dependent Variable: HurdleRate

Fig. 10: Collinearity Statistics

Descriptives				Statistic	Std. Error
Studentized Residual	Mean			-2.887E-3	...
	95% Confidence Interval for Mean	Lower Bound		-1.813E-1	
		Upper Bound		...	
	5% Trimmed Mean			-7.721E-3	
	Median			...	
	Variance			1.000	
	Std. Deviation			...	
	Minimum			-1.48686	
	Maximum			2.21482	
	Range			3.70168	
	Interquartile Range			1.92478	
	Skewness			-.011	.218
	Kurtosis			-1.183	.433

Tests of Normality						
	Kolmogorov-Smirnov ^a			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
Studentized Residual	.126	123	.000	.930	123	.000

a. Lilliefors Significance Correction

Descriptives				Statistic	Std. Error
Standardized Residual	Mean			-9.03E-14	...
	95% Confidence Interval for Mean	Lower Bound		-1.740E-1	
		Upper Bound		...	
	5% Trimmed Mean			-5.288E-3	
	Median			...	
	Variance			.951	
	Std. Deviation			...	
	Minimum			-1.44869	
	Maximum			2.15019	
	Range			3.59888	
	Interquartile Range			1.87706	
	Skewness			-.003	.218
	Kurtosis			-1.176	.433

Tests of Normality						
	Kolmogorov-Smirnov ^a			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
Standardized Residual	.121	123	.000	.932	123	.000

a. Lilliefors Significance Correction

Fig. 11: Normality Tests of Standardised and Studentised Residuals

As we see from Fig.11, both the standardised and studentised residuals fail in the normality tests with p values very much less than 0.01. Hence, we cannot apply multiple linear regression. Instead we can proceed towards application of robust regression which is robust to violations of assumptions of homoscedasticity and normality. It will also take care of any outliers. Applying robust regression and using default 'bisquare' weighting function in MATLAB, we get the results as displayed in Fig. 12.

```
b =
    0.1985
    0.0774
    0.2838
    0.0006
   -0.0126
    0.0032
    0.1429
```

Fig.12: Robust Regression Output from MATLAB using 'bisquare' weighting function

```
mdl =

Linear regression model (robust fit):
y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6

Estimated Coefficients:

```

	Estimate	SE	tStat	pValue
(Intercept)	0.19854	0.004262	46.584	6.1851e-77
x1	0.077433	0.01661	4.6618	8.4416e-06
x2	0.28381	0.053807	5.2746	6.2528e-07
x3	0.00056314	0.00024855	2.2657	0.025326
x4	-0.012563	0.00018567	-67.661	4.6015e-95
x5	0.0031563	0.00092847	3.3995	0.00092636
x6	0.14292	0.026463	5.401	3.5752e-07

```

Number of observations: 123, Error degrees of freedom: 116
Root Mean Squared Error: 0.00251
R-squared: 0.986, Adjusted R-Squared 0.986
F-statistic vs. constant model: 1.4e+03, p-value = 1.17e-105

```

Fig.13: Robust Regression Statistics of applied model

As we see from Fig.13, Adjusted R squared value is 0.986 which means 98.6% of the variance of hurdle rate is explained by our independent variables. Also, p value of F Statistic is 1.17e-105 which is very much smaller than 0.05. Hence, the fit of our overall model is good. This means that there is a very less chance of all the coefficients to be zero at the same time. In case of robust regression, due consideration has to be given to t and p values of coefficients. All coefficients with t values closer to 2 and p values closer to 0.02 imply insignificance of that particular coefficient. Hence in this case coefficient of x3 or 'risk factor' is insignificant. Rest all coefficients are significant.

```
P =
    0.1643

DW =
    1.9771
```

Fig.14: Durbin-Watson Statistic

As seen in Fig.14, the Durbin-Watson statistic is 1.9771 which is close to 2. Hence, we can assume that residuals for robust regression are not auto correlated. Even if the picture is clearer now, few other details need to be determined. The relative significance of each independent variable and the total information provided by them for estimating hurdle rate is still not known. Running stepwise regression will help us to know the relative importance of

each individual independent variable. Running stepwise linear regression on SPSS yielded the results as shown in Figs. 15 and 16.

Model Summary ^a					
Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Durbin-Watson
1	.988 ^a	.976	.976	.00328	
2	.990 ^b	.980	.979	.00301	
3	.992 ^c	.984	.984	.00267	
4	.993 ^d	.986	.985	.00252	
5	.994 ^e	.987	.987	.00240	
6	.994 ^f	.988	.987	.00235	1.968

a. Predictors: (Constant), ImportanceDiscount
b. Predictors: (Constant), ImportanceDiscount, WACC
c. Predictors: (Constant), ImportanceDiscount, WACC, DefaultPremium
d. Predictors: (Constant), ImportanceDiscount, WACC, DefaultPremium, Inflation
e. Predictors: (Constant), ImportanceDiscount, WACC, DefaultPremium, Inflation, Tenure
f. Predictors: (Constant), ImportanceDiscount, WACC, DefaultPremium, Inflation, Tenure, RiskFactor
g. Dependent Variable: HurdleRate

Fig.15: Stepwise Regression Results detailing relative significance of independent variables

Studying information contained in Fig.15 and 16 leads us to believe that most significant information is contained in importance discount which explains 97.6% of the variance in hurdle rate. Hence we can easily comprehend that hurdle rate is more or less decided once its importance

has been determined. WACC is the next most significant variable. Default premium, expected inflation, tenure, and risk factor follow WACC in that order of significance. Risk factor is the least important variable as observed previously also.

Findings

The various findings of the study are as listed under:-

1. Importance discount is the most important independent variable included in our study. It carries the most significant information required to develop a hurdle rate. It is negatively correlated with hurdle rate. It means when a project is thought to be strategically more important, the hurdle rate is reduced accordingly to accommodate the project.
2. WACC is the second most important independent variable in our study. Actually, it should be theoretically called the most important variable which roughly decides the hurdle rate. Importance discount only helps in fine tuning the hurdle rate.
3. Default premium is the third most important variable in our study. Taking a look at the values for de-

Excluded Variables ^f							
Model		Beta In	t	Sig.	Partial Correlation	Collinearity Statistics	
						Tolerance	Minimum Tolerance
1	WACC	.065 ^a	4.842	.000	.404	.944	1.060
	DefaultPremium	.063 ^a	4.431	.000	.375	.873	1.145
	RiskFactor	.053 ^a	3.027	.003	.266	.611	1.638
	Tenure	.007 ^a	.489	.626	.045	.964	1.037
	Inflation	.058 ^a	4.179	.000	.356	.921	1.086
2	DefaultPremium	.072 ^b	5.808	.000	.470	.859	1.164
	RiskFactor	.031 ^b	1.788	.076	.162	.551	1.813
	Tenure	.020 ^b	1.516	.132	.138	.927	1.078
	Inflation	.046 ^b	3.425	.001	.300	.876	1.141
3	RiskFactor	.028 ^c	1.844	.068	.167	.551	1.815
	Tenure	.010 ^c	.821	.413	.075	.906	1.104
	Inflation	.045 ^c	3.887	.000	.337	.876	1.141
4	RiskFactor	.036 ^d	2.492	.014	.225	.542	1.844
	Tenure	.048 ^d	3.682	.000	.322	.645	1.550
5	RiskFactor	.034 ^e	2.448	.016	.222	.541	1.848

a. Predictors in the Model: (Constant), ImportanceDiscount
b. Predictors in the Model: (Constant), ImportanceDiscount, WACC
c. Predictors in the Model: (Constant), ImportanceDiscount, WACC, DefaultPremium
d. Predictors in the Model: (Constant), ImportanceDiscount, WACC, DefaultPremium, Inflation
e. Predictors in the Model: (Constant), ImportanceDiscount, WACC, DefaultPremium, Inflation, Tenure
f. Dependent Variable: HurdleRate

Fig.16: Excluded Variables Study in Stepwise Linear Regression

fault premium, we see that only values of 2% or 3% are utilised. Hence, these almost standardised values provide enough safety cushions for the probable error in setting of the hurdle rate.

4. Expected inflation is our next significant variable. The probable reason, why it is placed lower in the significance list is that it is already factored while preparing power sale agreements. Once, it is accepted that inflation could be an issue; sufficient incorporations are made in the power sale agreement.
5. Tenure appears next on our list. It is placed lower in significance list because like expected inflation, it is already factored while preparing power sale agreement. Another factor is that while a long tenure implies uncertainty, it always provides you with a chance to incorporate some new technology to reduce costs and earn higher income.
6. Unconventional risk comes lowest in our significance list. While all of the systematic risk is factored in a project's WACC, unconventional risk is an unsystematic risk which doesn't provide you with a chance to earn enhanced incomes. Hence, having a higher unconventional risk reduces a project's importance discount and even factored in debt part in WACC as well as in slightly increased default premium, e.g. banks would usually charge a higher interest rate for a project with higher unconventional risk. Finally, Unconventional risk values carry little separate valuable information and hence the result.

Recommendations

The various recommendations for the power projects are as listed under:-

1. Strategic importance of a power project should be carefully studied before analyzing WACC and hurdle rate. Any similar power project with equal configurations should be studied carefully. Several factors should be developed for assessing strategic importance of a project. Ratings should be done on each of these factors and finally comprehensive report with strategic importance value should be utilised for arriving at a hurdle rate.
2. Special consideration should be given to WACC of strategically important projects. Effort should be made to reduce cost of debt burden of these projects as cost of equity is more or less fixed. Better risk

handling measures, large size, reliable third party, fuel supplies, and other factors should be utilised to better work out interest charges from banks.

3. Default premium is more or less fixed in arriving at hurdle rates. It should be ideally set at 3% if there are no serious issues of concern. If issues do crop up or are estimated to be present, then default premium should be fixed at 4%. It should be set higher than 4% as this may lead to rejection of few profitable projects.
4. Expected inflation shouldn't be factored much in arriving at a hurdle rate. Instead efforts should be made to hedge the adverse effects of inflation on future cash flows. This can be done by incorporating suitable arrangements in power sale agreements.
5. Similar to expected inflation, tenure also shouldn't be factored much in arriving at a hurdle rate. Instead efforts should be made to work up against the negative fallouts and uncertainties of a long tenure by incorporating suitable arrangements in power sale agreements. In addition, suitable arrangements for incorporating newer technology of producing power should be made in these agreements. This will provide projects with a new tool to reduce cost in future as well as protect against closure of power projects using old technology and fuel due to international norms.
6. Unconventional risk seems to convey the least valuable information and hence shouldn't be factored much in arriving at a hurdle rate. Nevertheless, it should be studied carefully for avoiding possible losses. As bearing unconventional risk doesn't lead to greater returns, hence suitable ratings should be carefully done on each of the strategic importance factors as discussed previously in strategic importance discount section. In addition, certain other provisions should be made so as to bear the least possible interest payments.

Limitations

The various limitations of the study are as listed under:-

1. The number of power generation projects involved in the study is limited as their number is limited and all projects did not reveal all information.
2. Only selected variables were considered for the study.

3. Tax rate is assumed to be same for all the power projects and hence its effect on WACC is considered same.
4. Effect of government aids available to any project has been ignored.
5. Hurdle rate values have been arrived at by values expected by higher level directors, independent analysts, and other experts.
6. Weights applied to each observation for their inclusion is assumed to be known before hand.

Conclusion

Hurdle rates have been generally placed closer to WACC values. Moreover, few organisations even relate the two terms to mean the same thing or as interchangeable. This has led to application of inappropriate values of hurdle rates in the past. Hurdle rates have been arbitrarily applied based on certain rules of thumb by higher level directors in consensus with banks and other consulting experts. Application of regression has developed a cognizable relationship among the variables. The final equation to be applied to arrive at a hurdle rate is as given below:-

$$\text{Hurdle Rate} = 0.1985 + 0.0774 * \text{WACC} + 0.2838 * \text{Default Premium} + 0.0006 * \text{Unconventional Risk Factor} - 0.0126 * \text{Strategic Importance Discount} + 0.0032 * \text{Tenure} + 0.1429 * \text{Expected Inflation} \quad (2)$$

where

WACC is expressed as an annual percentage,

Default Premium is expressed as a percentage added to WACC,

Unconventional Risk Factor is expressed as ordinal values denoting severity of unconventional risk on scale of 0 to 10 where 10 denotes maximum risk,

Strategic Importance Discount is expressed as ordinal values denoting relative strategic importance of the project on a scale of 0 to 10 where 10 denotes maximum importance,

Tenure is expressed as number of decades,

Expected Inflation is expressed as a fraction and not as a percentage

Strategic importance discount should be used most carefully and its value scientifically determined. Same should be the case with WACC. Maximum flexibility should be taken in fixing the value of unconventional risk factor which is of lowest significance in our study. In fact, its value can be slightly increased to be doubly sure of including all risk factors. Values of expected inflation should be taken from reliable government sources. Default premium values should be kept in the band of 2% to 3% depending on the situation as described before. Flexibility can be practiced in fixing of tenure due to the low value of coefficient as well as comparatively lower significance of this variable.

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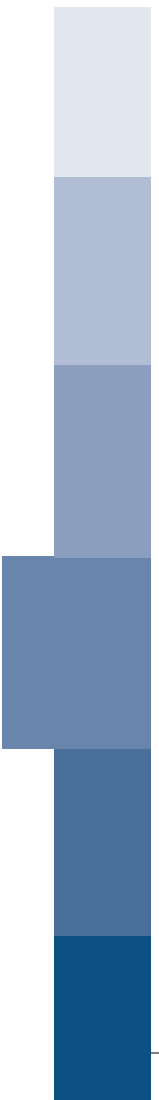
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